DATA FUSION III: Estimation Theory

Date: March 30, 2006**Time:** 5:00 – 7:30 PM

Location: B-300-2-3 (AAR-400) (Main Building, 2nd floor, near freight elevators)

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Credits: 1 **Course Code:** 0901-501-05 **Registration Number:** 13460

Today's topics:

1 Questions Received

1.1 The Snake-Oil Tracker

Some questions on Lecture 4 (March 16, 2006) – Snake Oil Tracker

Questins

1. Eqn 2.2: Is there one measurement y which is the position x1 ? Where does the value of the variance y, y come from ? Is the variance of the state variable x1 in the state covariance matrix P, equal to the variance of y, y ?

2. Eqn 2.4: How do we get the terms for the measurement covariance matrix ?

3. Eqn 2.8: Does h(x) = 1 ? How do we get H = [1 0] ?

4. Eqn 2.9: What happens to the measurement covariance R ?

5. What does this quote mean: "the Kalman gain is unmodified from the minimum variance solution" ?

6. Eqn 2.14: Is the variance of the sum (difference) of two random variables equal to the sum of their variances ? Is independence required ?

7. Eqn 2.15: How are the values of the diagonal elements of the plant noise covariance matrix, q11 and q22, obtained ?

Answers

1. Yes, the only measurement is the range. There are other variations on the Snake-Oil tracker that take Doppler, or use a chirped pulse; they were not presented here. Measurement variance estimation is a separate topic from Kalman filter formulation; it was discussed as the first topic on March 16. In Equation (2.2), yes, the variance of the state variable estimate xhat1 is in the covariance matrix is the variance of the measurement; that's what the first line of the equation says. After the next update, no, it's found through the covariance update.
2. See the first topic for March 16.
3. Note that h(x)=r, not 1, and dh(x)/dx = 1, dh(x)/d(xdot) = 0.
4. R is represented by the variance of y here.
5. The condition for validity of the short form for the covariance update is that the Kalman gain must be given by $K = \tilde{P} \cdot H^T \cdot \left(H \cdot \tilde{P} \cdot H^T + R \right)^{-1} = P \cdot H^T \cdot R^{-1}$ or other algebraically identical equivalent, as derived from minimizing the variance in the errors of $\underline{\hat{x}}$. That condition is met here, because our modifications of the Kalman equations are in the plant noise formulations, not in the Kalman gain equations.
6. When you compute the variance of the sum or difference of two uncorrelated random variables, the sign of either is irrelevant -- the variance is the sum of the variances. Independence is implied when you omit cross terms as $\rho = 0$ is

implicit. You always have independence when the errors in the measurements are uncorrelated from measurement to measurement. That's a big IF, because sometimes people don't know the difference and use filtered data, such as the output of another tracker, as input to a Kalman filter and things don't work out. This blunder is not as rare as it should be, and is discussed on March 30, 6.2.2.c page 13, and elsewhere in Data Fusion III. To keep independence in errors in y, you need to use raw (unfiltered) data or add measurement differencing (modeling of the filtering process in the measurements) to the Kalman filter.

7. The adaptive plant noise coupling coefficients qc_{11} and qc_{22} are defined as part of the Kalman filter formulation – they are assigned by the designer or software architect. They are used to tune the tracker performance. You begin by estimating the mean value of the second term (equal to 1, the way it is written in Equation (2.15)) and adjust qc_{11} and qc_{22} to make the process noise what you would use without adaptive processing. A good start would be to make qc_{11} zero and qc_{22} model expected aerodynamic buffeting. The coefficient qc_{22} must not be zero, as discussed elsewhere.

2 What are the characteristics, advantages, disadvantages of the Kalman filter as compared to the alpha-beta tracker?

2.1 Alpha-Beta Filters

2.1.1 A-B Advantages

- Simplicity in theory, formulation, and implementation.
- Fixed or formula gain ($\beta = \alpha^2 / (2-\alpha)$) is not affected by jamming or interference.
- Design, care and tuning doesn't require understanding of error analysis.
- Alpha-beta trackers are not easily extended to 2D trackers; range-only or bearing-only; need two separate trackers.
- Alpha-beta trackers are not easily extended to support bias problems or other real-world situations; basically a bare-bones concept for simple problems that don't require optimality.

2.1.2 A-B Disadvantages

- Although good performance is often attained, optimality is not addressed and, if any understanding of potential performance is required, a Kalman or batch estimator must be used in parallel. Most alpha-beta trackers for critical applications are developed first as Kalman filters and the filter gain modeled after what is observed in the Kalman filter, so the alpha-beta tracker is most effectively designed as an approximation to a Kalman filter.

- Fixed or formula gain doesn't treat position and velocity states separately
- Adaptive extensions add complexity similar to that of Kalman filters without the advantages of variance analysis; including variance analysis turns it into a Kalman filter

2.2 Kalman Filters

2.2.1 Kalman Advantages

- Properly formulated, optimal or near-optimal performance is relatively simple to obtain
- Complexity of two-state trackers is similar to that of high-performance alpha-beta
- Easily formulated for 1-D infrastructure tracker, 2-D nonaccelerating target tracker, 3-D, etc.
- Easily switches modes from non-accelerating (two, four or six state) to maneuvering target model tracking
- Easily formulated to deal with biases and other common problems
- Extensions exist such as IMM, MHT, correlated measurement, measurement differenceing, etc. to handle real-world problems
- Related in obvious ways to batch estimators that sometimes need to be run in the tracker function architecture for bias estimation
- Provide a covariance matrix that lends itself to use for localization ellipsoid
- Covariance matrix provides basis for data fusion

2.2.2 Kalman Disadvantages

- A radar contact with essential zero process noise cannot be tracked as accurately with a Kalman filter as with a batch estimator because some process noise must be used in a Kalman filter target model to prevent total covariance collapse, even in a square root filer.
- Taking full advantage of their capabilities requires judgment and skill that may not be present, or its need recognized.
- Some applications do not require their capabilities or performance.
- There are a lot of bad ones out there, and their problems are blamed on the technology rather than the implementation or underlying data or hardware problems and issues; fixing or replacing them can be a hard sell.

- Even after all these years, some who don't understand them oppose their use because they don't feel comfortable with managing their development and deployment.

3 What are the meaning and significance of the Cramer-Rao bound? Is it the best possible? What limits its values? Is it asymptotic to the number of samples?

The Cramer-Rao bound is the minimum variance (or covariance) that can be obtained by any estimator from a given set of data. In Kalman filters it's a theoretical limit of accuracy. In some batch estimators (efficient, or asymptotically efficient), it's achievable. It's a fixed function of the data set and the definition of the state vector. If you change either, it will change. For example, if the data set is augmented with additional data, it will change.

4 What properties of estimators make them suitable for independent use and what properties make them suitable for fusion with other estimators?

4.1 Suitable for independent use

I presume that you mean off-line use, data analysis use, or for use as an analysis too.

Batch estimators can provide performance that essentially achieves the Cramer-Rao bound, and they also give that variance (or covariance matrix) to you. When you're doing something just once, there's no reason to settle for less.

Simplicity -- nothing is simpler than a batch estimator for estimating parameters that don't require process noise for their formulation -- you define the measurements, the states, an algebraic relationship between them, and the covariance of the measurement errors. The estimator requires that you find the partial derivatives of the algebraic functions of the measurements in terms of the states, then "turn the crank." Nothing is simpler; "simple" methods that use intermediate steps simply add complication, etc.

Incorporation of process noise in the state vector propagation makes Kalman filters suitable for optimal estimation in problems that require process noise.

Estimators that provide a covariance matrix of errors in the estimate are best for data fusion.

Estimators that can be augmented to account for biases and offsets or estimate them are best for data fusion, both for input to data fusion and for performing data fusion.