# Data Fusion III – Estimation Theory

# QUIZ

## Rules

### Please work alone

Please work alone, although you may ask me questions by email. I will reply when appropriate by email to all so that everyone has the same clarifications (and to avoid redundant questions by clarifying just once when needed).

### Open Book, Open Notes, Computer Encouraged

Use of the text, *Applied Optimal Estimation* by A. Gelb (editor), and the course handouts, including updates posted on http://rowan.jkbeard.com, is encouraged as references while working this quiz. Also, use of notes taken by yourself during class and while studying, and your homework and other notes by yourself are OK to use while working this quiz. You are encouraged to use your computer to help with problems or to work this quiz.

### Turning In the Quiz

The quiz is due before close of business on Thursday, February 23, 2006. You may

- Send a file by email to beard@rowan.edu or jkbeard@comcast.net
- FAX it to (609) 654-8751
- Make arrangement by email for me to pick up the quiz at WJHTC

### Format of your Work

Your work must be legible and your reasoning must be clear. Use complete sentences in any explanations when you write out an answer or explanation. References must include the name of the book, page numbers, heading numbers, and, if relevant, equation numbers. Additional information such as "third paragraph" and a quote of a word or two such as "that begins with 'A sequence of measurements…'" will help me to locate your reference.

If you use handwriting, please scan to a GIF, TIFF, JPG, or PDF file (PDF preferred) and e-mail me that file, or use FAX to the number I gave above.

### Percentage Credit

There are four problems, each with 25% credit. Each has two or three parts. Credit for each part is given on the problem.

# Problem 1

## Part A (40%)

Define these terms and their place in estimation theory:

- A *statistic*
- An *estimator*
- A *confidence limit*
- An *unbiased estimator*
- A *consistent estimator*
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## Part B (40%)

- Take a few minutes to explain what a *sufficient estimator* is.
- Define an *invariant* estimator.
- What is an *efficient* estimator?

## Part C (20%)

The Kalman update is an estimator that uses the extrapolated state vector $\underline{\tilde{x}}$ and the measurement data vector $\underline{y}$ to compute an estimator $\underline{\hat{x}}$ for the state vector $\underline{x}$,

$$\underline{\hat{x}} = \underline{\tilde{x}} + K \cdot \left( \underline{y} - \underline{h}\left( \underline{\tilde{x}} \right) \right)$$

where $K$ is the Kalman gain,

$$K = \tilde{P} \cdot H^T \cdot \left( H \cdot \tilde{P} \cdot H^T + R \right)^{-1}$$

$\underline{y}(\underline{x})$ is an algebraic model of the measurements $\underline{y}$ in terms of the states $\underline{x}$, and $H$ is the gradient of $\underline{y}$ with respect to $\underline{x}$

$$H = \left. \frac{\partial \underline{h}\left( \underline{x} \right)}{\partial \underline{x}} \right|_{\underline{x} = \underline{\tilde{x}}}$$

and $\tilde{P}$ is the covariance of the extrapolated state vector $\underline{\tilde{x}}$. The covariance of the updated state vector estimate $\underline{\hat{x}}$ is

$$P = \left( I - K \cdot H \right) \cdot \tilde{P} \cdot \left( I - K \cdot H \right)^T + K \cdot R \cdot K^T$$

Under what conditions is a Kalman update an efficient estimator of the state vector?

*HINT:* Use the interpretation of the Kalman update as a maximum likelihood estimator.

# Problem 2

## Part A (50%)

.Define these three estimator techniques and briefly describe the distinctions between them:

- Maximum likelihood estimator
- Maximum *a posteriori* estimator
- Bayesian mean

## Part B (50%)

Given an unknown state vector $\underline{x}$ and a set of measurements $\underline{y}$, and the conditional probability density function $p\left(\underline{y}\middle|\underline{x}\right)$, write the equation for the general form for:

- The *likelihood function*
- The *log likelihood function*
- The *likelihood equation*
- The *Cramer-Rao Bound*

# Problem 3

## Part A (50%)

Given a set of $M$ samples of Gaussian random variables that have a mean of $m$ and a variance of $\sigma^2$, we will write down a maximum likelihood estimator (MLE) for the mean. The measurements are of the form

$$y_i = m + v_i$$

where the measurement noises $v_i$ are Gaussian random variables with zero mean and variance $\sigma^2$. Write down

- The likelihood function.
- The log likelihood function.
- The likelihood equation for $m$.
- The estimator for $m$.
- The variance of the estimator for $m$.

Use the Cramer-Rao bound as your estimator for the variance of $m$.

## Part B (50%)

Modify the estimator for Part A to include the variance $\sigma^2$. Using the log likelihood function from Part A, write down

- The likelihood equation for $\sigma^2$.
- The estimator for $\sigma^2$.
- The variance of your estimator for $\sigma^2$.

Use the Cramer-Rao bound as your estimator for the variance of $\sigma^2$.

# Problem 4

Building a radar tracker, "Care and Feeding of Radar Trackers," is the problem we will address here.

## Part A (50%)

When an estimate of SNR is available from any means, write down a working equation for the variance of a measurement. Then, give

- The lowest possible measurement variance for very high SNR
- The measurement variance for zero SNR.
- The term $c_x$ for moderate SNR in the form

$$\sigma_y^2 \approx \frac{c_x}{SNR}$$

## Part B (50%)

Use the equations for the "Snake-oil" two-state tracker presented on March 16 in this part. Using the form for $\sigma_y^2$ for moderate SNR from Part A, write

- The equation for the Kalman gain in terms of the SNR.
- The equation for the updated state covariance $P$ in terms of the SNR.
- The ratio of the determinants of the extrapolated and updated covariance matrices in terms of the SNR.

# Problem 5

In this problem you will be asked for your judgement as a system engineer. Your grade will be based on your reasons as much as your conclusions.

## Part A (50%)

You are asked to provide a preliminary recommendation for the configuration of a radar tracker for a new radar, not an ASR-9 or DASR but a radar that is offered for networking by another Agency. This new radar will provide azimuth tick information, plus time tag information, and horizontal monopulse azimuth data. SNR will be available but not measurement variances. The SSR data is present, and the SSR antenna is mounted on top of the PSR antenna. The azimuth bias of this radar is unknown but uniform over 360 degrees – that is, azimuth bias is not a function of aircraft azimuth. In addition to aircraft angle, aircraft range rate information is available as Doppler information. The range of this radar for transport aircraft at altitude is 120 nautical miles. It is not located near an airport but air traffic routes pass over the coverage area of the radar.

The aircraft environment at this radar location is described as sparse to moderate, with no more than 10 radar contacts. The output of this radar is used to supplement ARSR radars but is not used to merge with data of radars that track the same aircrafts simultaneously. You need to make recommendations based on the following alternatives:

a) Would you recommend a Kalman filter or an alpha-beta filter? Why?
b) Do you need to estimate an additional azimuth bias state? Why?
c) Do you recommend that IMM be incorporated into the tracker design? Why?
d) Do you recommend an MHT layer on the tracker for track-before detect? Why?
e) If you estimate azimuth bias, would you estimate is with the tracker or add a batch estimator specifically for estimation of azimuth bias? Why?

## Part B (50%)

We have a radar identical to the one in Part A but at a different site. This site is near an airport and the new radar’s coverage area overlaps two existing ASR radar coverage areas and some ARSR coverage areas. Data from all radars that detect a particular aircraft will be fused, and a central C2 database will support an ATC facility at the airport. The aircraft environment at this radar location is described as dense, with 100 to 300 aircraft in its coverage area. Data from existing radars is available for estimating azimuth bias. You need to make recommendations based on the following alternatives:

a) Would you recommend a Kalman filter or an alpha-beta filter? Why?
b) Do you need to estimate an additional azimuth bias state? Why?
c) Do you recommend that IMM be incorporated into the tracker design? Why?
d) Do you recommend an MHT layer on the tracker for track-before detect? Why?
e) If you estimate azimuth bias, would you estimate is with the tracker or add a batch estimator specifically for estimation of azimuth bias? Why?