DATA FUSION II, Radar Trackers

Date: January 26, 2006 **Time:** 5:00 – 7:30 PM
Instructor: Dr. James K Beard
Credits: 1 **Course Code:** 0901-504-04 **Registration Number:** 13459

Today's topics:

1 Course Roadmap

1.1.1 Rationales

- Our objective is an understanding of:
  - Software radar trackers in FAA context, present and future
  - Tracker requirements and performance
  - Software (hierarchical) block diagrams
  - Tracker functional data flow charts and data definitions
  - Requirements of users of trackers
- Radar trackers are based on alpha-beta filters and Kalman filters.
- Alpha-beta and Kalman filters are based on linear system theory and statistics
- Linear system theory is the use of vectors and matrices in simple system models

1.1.2 What Collateral Benefits Will This Knowledge Provide?

- Team Participation and Project management
  - Knowledge and understanding of how system performance requirements map to track accuracy requirements, and vice versa
  - Importance and implications of different requirements and how to trade them off

- Understanding of Monte Carlo simulations
  - Why and when they are used
  - Their limitations
  - How to design, use, and interpret them
- Design of experiments
  - What an experiment setup is, and why it is done that way
  - How to define confidence levels of expected experiment outcomes
  - How to interpret outcomes of experiments

1.1.3 Flow of the Course Topics

- Review of the system engineering process
- Elementary linear algebra, to support the linear systems basis of trackers
- Review of probability and statistics, to support the error analysis in Kalman filters and to support the use and understanding of Monte Carlo simulations for evaluation of tracker performance
- A short introduction to linear system theory
  - How linear system theory models the movements of radar contacts
  - How linear system theory models the mapping of errors in the tracker
  - Noise and errors in linear systems
- How to use tracker measurements to update an alpha-beta tracker
- How to use tracker error analysis to minimize tracker errors
- Simple application of what we have already learned to use trackers for data fusion
  - Problems and issues in data fusion with trackers, and how to deal with them
- Software architectures of trackers
  - Tracker software (hierarchical) block diagrams
  - Tracker functional block diagrams (data flow diagrams)

2 Examples of Simple Tracker Methods

2.1.1 Alpha-Beta Tracker

Notation: Position at time t_i is x_i, velocity at time t_i is v_i. Ovetilde (tilde over the character) is the estimate extrapolated from the last update, and overhat (carat over the character) is the updated estimate.
At the first measurement, before another is available, the measurement is used::

$$\begin{aligned}\hat{x}_1 &= y_1 \\ \hat{v}_1 &= 0\end{aligned} \tag{2.1}$$

The position and velocity estimates are defined at the second measurement:

$$\begin{aligned}\hat{x}_2 &= y_2 \\ \hat{v}_2 &= \frac{y_2 - y_1}{t_2 - t_1}\end{aligned} \tag{2.2}$$

Extrapolation from last update (needed to prepare values for last update):

$$\begin{aligned} T_i &= t_i - t_{i-1} \\ \tilde{x}_i &= \hat{x}_{i-1} + \hat{v}_{i-1} \cdot T_i \\ \tilde{v}_i &= \hat{v}_{i-1} \end{aligned} \tag{2.3}$$

Update using new data y_i:

$$\begin{aligned} \hat{x}_i &= \tilde{x}_i + \alpha \cdot \left(y_i - \tilde{x}_i \right) \\ \hat{v}_i &= \tilde{v}_i + \frac{\beta}{T_i} \cdot \left(y_i - \tilde{x}_i \right) \end{aligned} \tag{2.4}$$

2.1.2 Relationship between α and β

The most commonly used value of β is

$$\beta = \frac{\alpha^2}{2 - \alpha} \tag{2.5}$$

The choice of α is determined by signal-to-noise ratio, time between updates, target behavior, etc.

2.2 Vectors as Lists of Numbers

Interactive lecture; please take notes

2.3 Arithmetic of Vectors and Matrices

Interactive lecture, please take notes. Topics include:
How to add, subtract, and multiply vectors and matrices
Division???

3 How to Use a High Level Language (HLL) for Analysis

The HLLs commonly available for analysis include

- Matlab
- The C Programming Language
- Object-oriented extended C (C++)
- Visual Basic
- Fortran 95/2003
- Pascal

HLLs used in system engineering include

- C
- Fortran
- Ada
- JOVIAL (Air Force only)

Higher level analysis tools include

- Mathematica
- Maple

Spreadsheet tools include

- Mathcad (free-field with equation typesetting, physical units tracking)
- Visual Basic macros in Excel

3.1.1 Simple Example

```
program long_ride
implicit none
character(len=40)::bot="bottles of beer",     &
  wall="on the wall",                          &
  down="Take one down and pass it around..."
integer::bottles

do

  bottles=99
  do while(bottles>0)

    print *,bottles,trim(bot)," ",trim(wall)
    print *,bottles,trim(bot)
    print *,trim(down)
    bottles=bottles-1
    if(bottles > 0) then
      print *,bottles,trim(bot)," ",trim(wall)
    else
      print *,"Ain't no ",trim(bot)," ",trim(wall),"!"
    end if
    pause
  end do

end do

end program long_ride
```

3.1.2 Second Example

NOTE: Avoid line wrapping in this text if you run this program.

```
program twlday
  integer, parameter, dimension (31) :: I = (/&
```

```
10,32,44,45,46,59,67,79,97,98,99,100,101,102,103,104,105,10
7,108,109,110,111,112,&
     114,115,116,117,118,119,120,121 /)
  integer, parameter, dimension (392) :: J = (/ &

8,21,2,26,16,13,2,14,17,24,25,26,2,25,13,11,22,21,12,2,26,1
6,17,24,12,2,14,22,27,24,26,16,2,14,17,14,26,&

16,2,25,17,30,26,16,2,25,13,28,13,21,26,16,2,13,17,15,16,26
,16,2,21,17,21,26,16,2,26,13,21,26,16,2,13,19,&

13,28,13,21,26,16,2,26,29,13,19,14,26,16,2,12,9,31,2,22,14,
2,7,16,24,17,25,26,20,9,25,2,20,31,2,26,24,27,&

13,2,19,22,28,13,2,15,9,28,13,2,26,22,2,20,13,1,26,29,13,19
,28,13,2,12,24,27,20,20,13,24,25,2,12,24,27,20,&

20,17,21,15,3,2,13,19,13,28,13,21,2,23,17,23,13,24,25,2,23,
17,23,17,21,15,3,2,26,13,21,2,19,22,24,12,25,2,&

9,4,19,13,9,23,17,21,15,3,1,21,17,21,13,2,19,9,12,17,13,25,
2,12,9,21,11,17,21,15,3,2,13,17,15,16,26,2,20,9,&

17,12,25,2,9,4,20,17,19,18,17,21,15,3,2,25,13,28,13,21,2,25
,29,9,21,25,2,9,4,25,29,17,20,20,17,21,15,3,1,&

25,17,30,2,15,13,13,25,13,2,9,4,19,9,31,17,21,15,3,2,14,17,
28,13,2,15,22,19,12,2,24,17,21,15,25,6,1,14,22,&

27,24,2,11,9,19,19,17,21,15,2,10,17,24,12,25,3,2,26,16,24,1
3,13,2,14,24,13,21,11,16,2,16,13,21,25,3,2,26,&

29,22,2,26,27,24,26,19,13,2,12,22,28,13,25,1,9,21,12,2,9,2,
23,9,24,26,24,17,12,15,13,2,17,21,2,9,2,23,13,9,&
     24,2,26,24,13,13,5,1 /)
  integer, parameter, dimension (12) :: K =
(/8,14,21,27,34,40,46,54,61,67,73,82 /), &
     L = (/13,20,26,33,39,45,53,60,66,72,81,89/), &
     M =
(/365,344,325,305,288,268,244,221,200,179,157,131/)
  integer :: n
  do n = 1, 12
   print*, achar( I((/J(1:7), J(K(n):L(n)), J(90:130),
J(M(n):) /)) )
  end do
end program twlday
```

4 Review of Probability and Statistics

4.1.1 What we need to do with probability and statistics

- Understand error analysis as applied to tracker performance evaluation
- Design, build, and understand a Monte Carlo simulation to evaluate tracker performance

Interactive lecture; please take notes. Topics will include:

- Definitions of probability
- Presenting the Probability Distribution Function, and the probability density function
- Basic principles of probabilities
- Noisy signals are random variables
- Means are averages, variances are RMS errors
- The Uniform distribution
- The Gaussian distribution

5 Homework

No homework this time.

Suggested study: Pose the alpha-beta tracker as a vector-matrix recursive filter:

$$\underline{x}_i = \begin{bmatrix} x_i \\ v_i \end{bmatrix}$$

$$\tilde{\underline{x}}_i = \Phi_i \cdot \hat{\underline{x}}_{i-1}, \;\; \Phi_i = \begin{bmatrix} 1 & T_i \\ 0 & 1 \end{bmatrix}$$

$$\hat{\underline{x}}_i = \tilde{\underline{x}}_i + K \cdot \left(y_i - h\left(\tilde{\underline{x}}_i\right)\right), \;\; K = \begin{bmatrix} \alpha \\ \dfrac{\alpha^2}{2-\alpha} \end{bmatrix}, \;\; h\left(\tilde{\underline{x}}_i\right) = \tilde{x}_i$$

Is there a simple way to write the function $h\left(\tilde{\underline{x}}_i\right)$ as a dot product?

6 One Final Note

Approximate Amazon sales ranks of popular tracking references as of January 26, 2006:

Book	**Approximate Sales Rank on Amazon.com**
Gelb	78,000
Bar-Shalom (2002)	85,000
Blackman and Popoli (2000)	225,000
Blackman (1986)	690,000