QUIZ

Rules

Please work alone

Please work alone, although you may ask me questions by email. I will reply when appropriate by email to all so that everyone has the same clarifications (and to avoid redundant questions by clarifying just once when needed).

Open Book, Open Notes, Computer Allowed

Use of the text, *Applied Optimal Estimation* by A. Gelb (editor), and the course handouts, including updates posted on http://rowan.jkbeard.com, is encouraged as references while working this quiz. Also, use of notes taken by yourself during class and while studying, and your homework and other notes by yourself are OK to use while working this quiz. You are encouraged to use your computer to help with problems or to work this quiz.

Turning In the Quiz

The quiz is due before close of business on Thursday, February 23, 2006. You may

- Turn it in to Dr. Beard at the beginning of Data Fusion III class
- Send a file by email to beard@rowan.edu or jkbeard@comcast.net or
- FAX it to (609) 654-8751

Format of your Work

Your work must be legible and your reasoning must be clear. Use complete sentences in any explanations when you write out an answer or explanation. References to Gelb must include page numbers, heading numbers, and, if relevant, equation numbers. Additional information such as "third paragraph" and a quote of a word or two such as "that begins with 'A sequence of measurements…'" will help me to locate your reference.

If you use handwriting, please scan to a GIF, TIFF, JPG, or PDF file (PDF preferred) and e-mail me that file, or use FAX to the number I gave above.

Some portions of the quiz will be two questions, and you will decide which to answer. Only one answer will be graded. When this is done, you will have a choice between an analysis question and an essay question. You don't have to decide until you hand in the paper, but you must designate which answer you are submitting on your quiz. I will ignore the other answer, if present.

Percentage Credit

There are four questions, each with 25% credit. Each has two or three parts. Credit for each part is given on the question.

Question 1

Using the results and notes on Example 1.0-1 for this question.

Problem statement

Three (3) sensors

- Each has a single noise measurement
- $z_i = x + v_i$
- Unknown *x* is a constant
- Measurement noises v_i have a mean of zero but are not necessarily uncorrleated
- Estimate of x is a linear combination of the measurements that is not a function of the unknown to be estimated, *x*
- The estimate will be unbiased – its ensemble mean will be equal to *x*

Part A (50%)

Show the equation for the gain vector $\underline{k}$ in terms of the covariance matrix R_v

Show the equation for the minimum mean square error J_{MIN} in terms of the covariance matrix R_v

Part B (50%)

Given that the measurement error covariance is

$$R_v = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}$$

Show the explicit expressions for the elements of the gain vector k_1, k_2, and k_3

Show the explicit expression for the minimum mean square error J_{MIN}

Problem 2

Use the handouts for Gelb's Problem 2-5 as a reference in working this problem.

Problem statement

Part A (30%)

Show that the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

satisfies the polynomial equation

$$A^2 - 4 \cdot A - 5 \cdot I = 0$$

Part B (20%)

Find the eigenvalues of the matrix *A* of Part A.

Part C (50%)

Using the Cayley-Hamilton theorem, find the eplicit forms for $a_1(t)$ and $a_2(t)$ in the equation

$$\exp\left(A \cdot t\right) = a_1\left(t\right) \cdot I + a_2\left(t\right) \cdot A$$

in terms of the eigenvalues λ_1 and λ_2. Then, give it explicitly in terms of the numbers for the eigenvalues that you found in Part B.

Problem 3

Use the notes from the lectures to work this problem

Problem Statement

We are defining the software functions for the data association problem for a tracker to be used in a dense target environment – a lot of airplanes close together, at different altitudes and flying in different directions with not a lot of ground distance between them. We are considering three cost functions for use in thresholding and tie-breaking in the data association function:
number:

Hyperpolygon

$$J_{HP} = \sum_{i=1}^{M} \frac{\left| y_i - h_i\left(\underline{\tilde{x}}\right)\right|}{\sqrt{\sigma^2\left(y_i\right) + \sigma^2\left(h_i\left(\underline{\tilde{x}}\right)\right)}} < thr_{HP}$$

Hypercube

$$J_{HC} = \underset{1 \le i \le M}{\text{Max}} \left(\frac{\left| y_i - h_i\left(\underline{\tilde{x}}\right)\right|}{\sqrt{\sigma^2\left(y_1\right) + \sigma^2\left(h_i\left(\underline{\tilde{x}}\right)\right)}} \right) < thr_{HC}$$

Hypersphere (Bhatacharya distance)

$$J_{HS} = \left(\underline{y} - \underline{h}\left(\underline{\tilde{x}}\right)\right)^T \cdot \left(H \cdot \tilde{P} \cdot H^T + R\right)^{-1} \cdot \left(\underline{y} - \underline{h}\left(\underline{\tilde{x}}\right)\right) < thr_{HS}$$

Problem 3 (Continued)

Part A (50%)

We have two measurements from a radar, range and azimuth. Show the role of the correlation between these in the equation for the Bhatacharya distance. For simplicity, use the form

$$J_{HS} = \underline{\Delta x}^T \cdot P_E^{-1} \cdot \underline{\Delta x}$$

and use this equation for P_E :

$$P_E = \begin{bmatrix} \sigma_{x1}^2 & \sigma_{x1} \cdot \sigma_{x2} \cdot \rho_{12} \\ \sigma_{x1} \cdot \sigma_{x2} \cdot \rho_{12} & \sigma_{x2}^2 \end{bmatrix}$$

Show the expression for J_{HS} expanded into its scalar terms, not as a quadratic form. Show the expanded expression again with ρ_{12} taken as zero. Explain the importance of ρ_{12} when the variance of one component of $\underline{\Delta x}$ is much larger than the other, and the correlation in the errors ρ_{12} is high (close to +1 or -1).

Part B (50%)

We have data from two radars and we want to merge the track files. We must associate track files from one radar with track files from the other radar. This operation is identical to the problem of association of radar returns to track files, except that state vectors from the first radar replace the measurement vector in the operation.

The state vectors from both radars as provided to the track file merging function are latitude, longitude, and altitude, plus velocity North, velocity East, and climb rate. Thus we have six states in the state vectors.

Evaluate the use of the hypercube or hyperpolygon instead of the hypersphere (Bhatacharya distance) in defining the cost function for association of these radar tracks. Use this as a rationale for requiring that the entire covariance matrix be provided by each radar, instead of simply the variances of each state. State this rational as simply as possible, and use as few sentences as possible.

Problem 4

Use the handouts and study guide for the Kalman filter equations to work this problem.

For part B, only one of the alternative questions will be graded. Indicate which you are submitting, even if you provide a response to only one alternative.

Problem statement

Give the Kalman filter equations outlined below and describe each term in them.

Part A (50%)

Give the meaning of

- The state vector $\underline{x}$,
- Its extrapolated value $\tilde{\underline{x}}$, and
- Its estimated value $\hat{\underline{x}}$.

Give the Kalman filter equations as defined in the study guide for the following quantities. Explain each term.

1. The measurement model; the equation beginning with $\underline{y} = \ldots$
2. The state vector extrapolation model; the equation beginning with $\dot{\underline{x}} = \ldots$
3. The covariance extrapolation approximation; the equation beginning with $\tilde{P} \approx \ldots$
4. The Kalman gain; the equation beginning with $K = \ldots$
5. The state vector update; the equation beginning with $\hat{\underline{x}} = \ldots$
6. The covariance update; any of the equations beginning with $P = \ldots$ or $P^{-1} = \ldots$

Part B (50%)

Submit only one of the two alternatives below. Designate your reply "Alternative 1" or Alternative 2." Only responses for the alternative that you designate will be graded.

Alternative 1

Give the form for the covariance update equation that you would use when first blocking out a Kalman filter in software for testing and simulation. Explain why you would use that form as opposed to each of the other alternative algebraic forms for the covariance update.

Alternative 2

Use the Matrix Inversion Lemma (as given on the web site http://rowan.jkbeard.com on the Files page) to show that the Joseph stabilized form is equal to the normal form when the Kalman gain K is equal to the optimum value.