ATEP SYS12525

Day 3

Radar Trackers and Applications for SAADS

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Radar Trackers and Applications for SAADS

November 2, 1999

Topic 15: The Data Association Problem

Sensor Systems Engineering for the 21st Century

Statistical Basis of Association

- Extrapolate States and Covariance to Time of Candidate Hit
- Compute Candidate Update Error and Test Statistic t

$$\underline{e} = \underline{y} - \underline{h}\left(\underline{\tilde{x}}\right)$$

$$t = \underline{e}^T \cdot \left[H \cdot \tilde{P} \cdot H^T + R\right]^{-1} \cdot \underline{e}$$

- Use Smallest “t” to Select “Winner”
- Save <u>e</u>, Matrix Inverse for Kalman Update

Association Strategies

- Object: To Limit Computational Requirements
- Strategy
  - Use Prefiltering or Pruning of Obvious Cases
    - » Simple Inequalities on Azimuth, Range, Doppler
    - » Select Most Accurate Measurements to Use First
  - Use Simpler Computations (With Care)

Alternative Association Computations

- Hyperpolygon

$$\sum_{i=1}^{M} \frac{\left| y_i - h_i\left(\underline{\tilde{x}}\right) \right|}{\sqrt{\sigma^2\left(y_i\right) + \sigma^2\left(h_i\left(\underline{\tilde{x}}\right)\right)}} < t_1$$

- Hypercube

$$\left| y_i - h_i\left(\underline{\tilde{x}}\right) \right| < t_2 \cdot \sqrt{\sigma^2\left(y_i\right) + \sigma^2\left(h_i\left(\underline{\tilde{x}}\right)\right)},\ 1 \le i \le M$$

Comparative Analysis

- Analyze the Alternative Methods
  - Chi-Square Test
  - Hyperpolygon
  - Hypercube
- Important to Understand
  - Comparative Behavior for M > 2
  - To Support Choice of Method

One Probability Distribution

- Chi-Square Test When Association is Correct:
  - Test Statistic t is Chi-Square with M Degrees of Freedom
  - Mean M, Variance 2M
  - Probability of Association Failure Controlled with Threshold
- Probability of False Association is Defined

Probability of False Association

- Scenario
  - Association Volume is
    - Ellipsoid in M-Space
    - Hyperpolygon in M-Space
    - Hypercube in M-Space
  - Distribution of False Alarms (And Some Incorrect Targets) is Uniform in M-Space
- Probability is Ratio of Association Acceptance Volume to Total Volume

Comparison of Association Volumes

- Hypersphere – Statistical Basis

$$V = \frac{\pi^{\frac{M}{2}}}{\Gamma\left(\frac{M}{2}+1\right)}$$

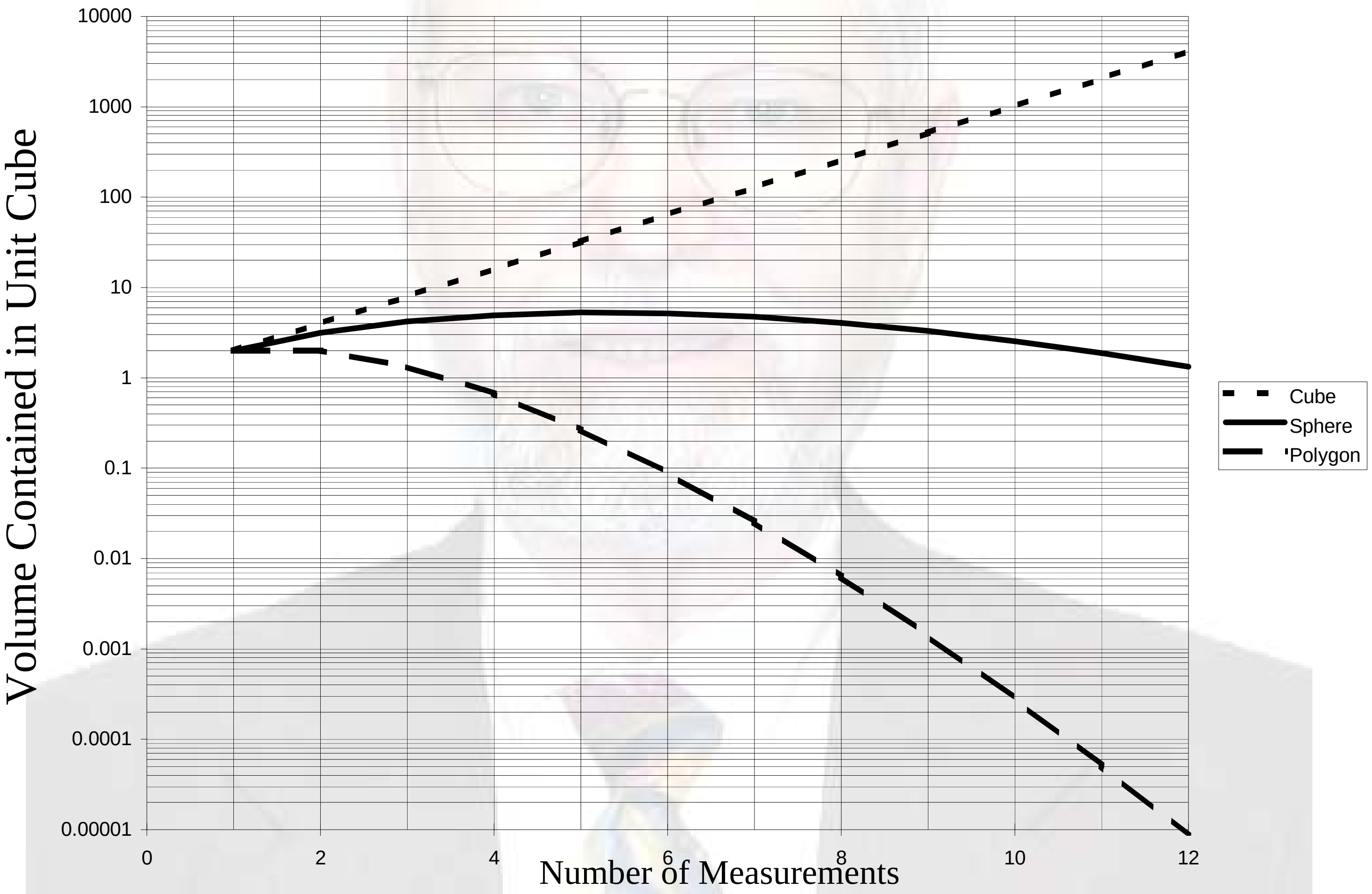
- Hyperpolygon – Contained Within Hypersphere

$$V_P = \frac{2^M}{\Gamma(M+1)}$$

- Hypercube – Contains the Others

$$V_C = 2^M$$

Comparison of Association Volumes



Complexity of the Association Problem

- Straightforward Association of K Detections with L Tracks Requires that K·L Comparison Computations Be Performed
- Each Comparison Computation Requires
  - Extrapolation of States and Covariance
  - Computation of Covariance of Filter Error
  - Computation of Chi-Square Statistic
- Association Is the Computational Requirements Driver for the Tracker Function

Design of the
Association Function

- Use Prefiltering to Prune Comparisons
  - Use “Sanity Check” Ranges on Measurements
  - Order the Measurements to Eliminate Most Non-Candidates
- Use Simple Nearest Neighbor Checks
  - When the Environment is Not Dense
  - Validation Only – No Critical “Tie Breaker” Situations
  - One, Two or Three Measurements
- Use the Chi-Square Test for
  - Critical Applications
  - Dense Environments
  - Breaking Ties
  - More than Three Measurements

Hypothesis Testing

- Simple Detection Sensors – Hypothesis Testing is at Signal Processor Output Only
- Typical Implementations – Some Track Files are Not Displayed if K misses in a Row or Other Criteria are Met
- Multiple Hypothesis Trackers – In a System Using an MHT, Signal Processor Threshold is Lowered and Both Signal Tracks and Noise Tracks are Formally Treated in the Tracker Design

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Topic 16: Multiply Hypothesis Tracker Concept

Sensor Systems Engineering for the 21st Century

Why an MHT

- Sensitivity
  - Sensor System Sensitivity Increase
  - Trades Off
    - » Processing Power
    - » System Sensitivity
- Very Dense Environment
  - Multiple Associations Necessary for Tracking
  - Use Statistics to Sort It All Out
- Robustness
  - It doesn’t break when the going gets tough

What Is an MHT

- MHTs are
  - Track-Before-Detect
  - Practical, Realizable
  - Core Raytheon Systems Technology
- Sensitivity Increased By Lowering Detection Threshold
  - “Sees” Weaker Targets
  - Allows Higher False Alarm Rate

Architecture of a MHT

- Lower Detection Threshold
- Initiate Tracks on False Alarms
- Bifurcate Tracks
  - Multiple associations
  - Extra tracks to allow for misassociations
  - Extra tracks to allow for different target
    - Types
    - Behaviors
- Report Tracks as Detections When
  - Score Function Exceeds Threshold
  - Operator or Command and Control Directs
- Drop Tracks When
  - Score Function Falls Below Threshold
  - Operator or Command and Control Directs

Conventional Sensor Subsystems

- Hypothesis Testing
  - – For Detection
  - – At Signal Processor Output
- Design
  - – High Threshold
  - – System False Alarm Rate Achieved Here
  - – All Detections Assumed to Be Targets

Typical Implementation

- First Hypothesis Testing
  - At Signal Processor Output
  - High Threshold, Low False Alarm Rate
- Quality of Track Indicators Used to Blank Unlikely Targets
- Use of Second Threshold
  - Track Files are Subjected to Hypothesis Testing
  - Hypotheses Allowed at Track File Level
    - Target tracks
    - Noise tracks
    - Misassociated tracks

Multiple Hypothesis Trackers

- Fundamental Advantages
  - First hypothesis testing at signal processor is “weak”
  - Detections are “probable targets” or “possible targets”
  - Provides low system false alarm rate in harsh clutter environments
- Performance Advantage
  - Robust performance with high target densities
  - System probability of detection can exceed probability of detection at signal processor output
  - System false alarm rate controlled as a parameter in the MHT trade space

MHT Discriminators

- Low Threshold, High False Alarm Rate at Signal Processor Threshold
- Sequential Likelihood Ratio Test Used as a Track File Quality Indicator
- Track Files are Formally Allowed with Multiple Hypotheses
  - Target Tracks
  - Noise Tracks
- Score Function Subjected to Formal Hypothesis Testing Supports System Detections
- Achieve Additional System Processing Gain as Determined by Improvement in ROC Curve Over That Possible at Signal Processor Input
- Sorts Out Properly Associated Tracks for C^2 and Display

MHT Performance Tradeoffs

- Data Latency
  - Often More Than One Hit Necessary for a System Detection
  - Latency Increases with Additional Gain Increase and Weaker Targets
- MHT Configuration Trade Space
  - Probability of Detection
  - Probability of False Alarm
  - Detection Latency
  - Processor Requirements

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Topic 16: The MHT Score Function: Sequential Likelihood Ratio

Sensor Systems Engineering for the 21st Century

Sequential Likelihood Ratio

- Sources
  - Blackman, pp. 258-260, 405, other.
  - Reid, D. B., “An Algorithm for Tracking Multiple Targets,” IEEE Trans. Aut. Cont., AC-24, Dec. 1979, pp. 843-854.
  - Alspach, D. L., and Lobbia, R. N., “A Score for Correct Data Association in Multi-Target Tracking,” Proc. 1979 IEEE Conf. CDC, Ft. Lauderdale, FL, Dec. 1979, pp 389-393.
  - Bottlik, I. P., “Multiple Hypothesis Tracking – An Implementation,” IDC 85IPB05-2, May 8, 1985.
  - Beard, J. K., “MHT Tracker Processing Requirements,” Hughes IDC 7341/JKB06, June 23, 1989 (Handout/reading for this class).

Rationale of MHTs

- Formalization of Traditionally *Ad Hoc* Track Quality Score Functions
- Use of Results of Statistical Analysis of the Resulting Score Functions to Determine the Statistics of Dropped Tracks
- Use of Results of Statistical Analysis of Score Functions to
  - Allow Tracking Noise
  - Perform System Detections
- Thresholding Score Function Performs System Detection
- The Track File Score Function is the Key

From Raytheon ATEP SYS12525 for SAADS Day 3, November 19, 1999

Score Function Statistics

- Fixed Thresholds for
  - System Detections
  - Dropping Tracks
- Thresholds Independent of
  - Other Track File Quality Indicators
    - » Number of Hits
    - » Number of Consecutive Misses
  - State of Track File

Sequential Probability Ratio Test

- Objective

  To Define a score function, and a statistical test based on this score function, which determines whether

  (a) A track file is based on data made up of target returns or

  (b) A track file is made up of false alarms.

- Statistical Principles
  - P(event) = P(event1 & event2 & ... & eventn) = P(event1)·P(event2)· ... ·P(eventn) (i.e., events are uncorrelated)
  - P(eventa)/P(eventb) = t is a test statistic for thresholding to decide whether event a or event b is more likely.

Statistical Principles (Continued)

- A Track File is the Result of n Detections which are Associated with a Single Track File and Used to Update it
- The Probability that a Data Point is a Result of a Target Return is Available from the Statistics of the Association Process
- The Probability That a Data Point is a False Alarm is Available from the Statistics of the Detection Process

Sequential Likelihood Ratio

- When
  - The Probability Law Governing the Distribution of the Observed Measurements is Continuous as Opposed to Discrete
  - The Same Arguments Used to Develop the Sequential Probability Ratio Test Result in Likelihood Ratios
- The Sequential Likelihood Ratio Score Function
  - Accumulate association statistics
  - Add terms accounting for multiple hypotheses

Formulating the Score Function

- Log Probability
  - Won’t Cause Overflow
  - Allows Addition and Subtraction for Sequential Probabilities
  - Natural with Most Distribution Functions
- Logarithm is Monotonic
  - Higher/Lower Probability, Higher/Lower Log Probability
  - Thresholding One is Equivalent to Thresholding the Other

Association Policy

- Every Association
  - Updates or Initiates a Track File
  - Multiple Associations with One Track File
    - » Allowed
    - » Each Association Spawns a Separate Track File
  - Track file coasted without association
- Bifurcation with Multiple Associations
  - Leads to Nomenclature “Multiple Hypothesis”
  - Hypotheses
    - » Hit is from right target
    - » Hit is from a false alarm from clutter or noise
    - » Hit is from the wrong target
    - » No hit this time

Other Features

- Vary With Implementation
- Track File Merging
  - Track File Association Techniques Decide
  - Data Fusion Techniques Used
  - May or May Not Be Selected in a Design
- Approximations and Simplifications
  - Score Function Computations
  - Validate with Analysis, Simulation, Flight Testing

Radar Trackers and Applications for SAADS

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Topic 17: Analysis and Design of an MHT

Sensor Systems Engineering for the 21st Century

The Score Function

- The Score Function is the Key to a Successful MHT Implementation
  - It’s the Enabling Technology
  - It Determines Your Tradeoff Space
- Keeps Track of Relative Likelihood
  - That a Track is of a Target
  - That a Track is of False Alarms
- Based on Probability Theory

Terms for Each Hit

- If There Is an Association
  - – P(Proper Association |Data as Measured)
  - – P(Improper Association | Data as Measured)
  - – P(False Alarm | Measurement Space)
- If There is No Association
  - – P(No Detection | SNR from Track File)
  - – P(False Alarm | Measurement Space)
- Computed as Log Likelihood

The First Hit

- Log Relative Sequential Probability
  - LRSP Initialized as

$$L_0 = \ln\left(SP_{NT}\left(\underline{y}\right)\right) - \ln\left(SP_{FA}\left(\underline{y}\right)\right)$$

  - Where
    - » SP_{NT} is
      - Specific probability of a new target
      - An estimate of the probability of a new target appearing in the measurement space where the new detection occurred
    - » SP_{FA} is the probability of false alarm in the same measurement space position

Conditional Probabilities

- Define hypotheses
  - – H0 – Target not present
  - – H1 – Target is present
- The specific probabilities are

$$SP_{NT}\left(\underline{y}\right) = p\left(\underline{y}\middle|H1\right)\cdot d\,\underline{y}$$

$$SP_{FA}\left(\underline{y}\right) = p\left(\underline{y}\middle|H0\right)\cdot d\,\underline{y}$$

- Where $p(\underline{y}|Hx)\cdot d\underline{y}$ is the product of the $p(y_i|Hx)\cdot dy_i$

The Log Probability Ratio – Subsequent Hits

- The Log Sequential Probability Ratio

$$dL = \ln\left(P_D \cdot p\left(\underline{y} \middle| H1\right) \cdot d\,\underline{y}\right) - \ln\left(SP_{FA} \cdot d\,\underline{y}\right)$$

- The Conditional Probability Density is

$$p\left(\underline{y} \middle| H1\right) = \frac{1}{(2\pi)^{\frac{M}{2}} \cdot |E|^{\frac{1}{2}}} \cdot \exp\left(-\tfrac{1}{2} \cdot \underline{e}^T \cdot E^{-1} \cdot \underline{e}\right)$$

- Where

$$M = \text{Number of Measurements}$$

$$\underline{e} = \underline{y} - \underline{h}\left(\underline{\tilde{x}}\right)$$

$$E = \text{Cov}\left\{\underline{e}\right\} = H \cdot \tilde{P} \cdot H^T + R$$

Final Form of LSPR Increment

$$
\begin{aligned}
dL = {} & \ln\left(P_D\right) - \ln\left(SP_{FA}\right) \\
& - \frac{M}{2} \cdot \ln\left(2\pi\right) - \frac{1}{2} \cdot \ln\left(\left|E\right|\right) \\
& - \frac{1}{2} \cdot \underline{e}^T \cdot E^{-1} \cdot \underline{e}
\end{aligned}
$$

The Log Probability Ratio – No Hit

- When No Detection is Associated with a Track File when a Target “Should” Have Been in the Field of View, We Use the Probability that the Target was Not Detected: 1- P_D
- UP_{FA} is the Probability that a False Alarm Appears in the Association Space

$$dL = \ln\left(1 - P_D\right) - \ln\left(1 - UP_{FA}\right)$$

The Probability of Association with a False Alarm

- Function of
  - False Alarm Probability Densities
  - Association Volume in the Measurement Space
- Definition of the Association Volume is Necessary
- Detection and False Alarm Probabilities are Weighted Association Volume Ratios

Analysis of the Score Function

- When a target is associated with the track file, the incremental log probability ratio is chi-square distributed with M degrees of freedom.
- An additional quantity in the score function is ln(|E|), which may also be a random variable. Blackman, pp. 335-336, describes an approximation sometimes used to find |E| for analysis purposes. In simulation, this is no substitute for using the final form for the actual Kalman filter used for updating track files.

Analysis of Score Function (Continued)

- When a false alarm is associated with the track file, the quadratic form in the association process is distributed according to

$$P\left(r^2 \le x\right) = \frac{x^{\frac{M}{2}}}{R_0^M}$$

when an association limit R_0^2 is used.

- When no detection is associated wit the track file, the incremental score function is fixed. However, the statistics of selection between the incremental score functions must also be taken into account when formulating the statistics of the score function

Practical Points

- The Mean of the Incremental score Function is Easily Found with Reasonable Approximations
- The Variance of the Total score Function over Several Frames is Small for a Given Hypothesis
- The Behavior of a score Function is Characterized by the Mean Change in the score Function per Frame.

Why an Association Limit?

- No Limit will Allow An Association Whenever there is a False Alarm
- A Large Value of the Quadratic Form will Result in an Impossibly Low Value of the Sequential Probability Ratio
- Result
  - No Impact on Performance over No Limit
  - Simplification of Software

A Value for the Association Limit

- A Reasonable Crossover

$$dL(\text{with hit}) = dL(\text{without hit})$$

- Equivalent to

$$\underline{e}^T \cdot E^{-1} \cdot \underline{e} < R_0^2$$

$$R_0^2 = 2 \cdot \ln\left(\frac{P_D}{(1 - P_D) \cdot SP_{FA} \cdot (2\pi)^{\frac{m}{2}} \cdot |E|^{\frac{1}{2}}} \right)$$

Interpretation

- The Limit is
  - – Likelihood of Correct Association is as Low as That of a False Alarm
  - – Allowing Associations Past This Limit is Equivalent Statistically to Associating with a False Alarm
  - – The Limit of Benefit to the Track File
- Some Trial and Error Variation is Appropriate

Mean Change in the Score Function per Frame

- Required Data
  - – The Probability of Detection P_D
  - – The Specific False Alarm Rate SP_{FA}
- Specific False Alarm Rate Known
- Probability of Detection
  - – Easily Estimated from SNR
  - – Random Variable
- Denote Estimated P_D as EP_D

Final Result

- Mean Change in score Function per Scan

$$Exp\left\{dL\middle|H1\right\} = \left(1 - P_D\right)\cdot\ln\left(\frac{1-EP_D}{1-UP_{FA}}\right)$$

$$+P_D\cdot\left(\begin{array}{l}\ln\left(\dfrac{EP_D}{SP_{FA}}\right)-\dfrac{M}{2}\cdot\ln(2\pi)\\ -\dfrac{1}{2}\cdot\ln\left(\left|E\right|\right)-\dfrac{M}{2}\end{array}\right)$$

- Effect of Association Limit Neglected

Score Function for Noise Tracks

- Update False Alarm Probability – Product of
  - Probability of False Alarm in Each Element in the Measurement Space
  - Volume of Measurement Space Accepted for False Alarms
- Volume of Measurement Space in Elements is

$$V_{VA} = \frac{\pi^{\frac{M}{2}} \cdot R_0^2}{\Gamma\left(\frac{M}{2}+1\right)} \cdot |E| \cdot \prod_{i=1}^{M} \frac{1}{(\Delta y_i)}$$

$$\Delta y_i = \text{ bin width for measurement i}$$

False Alarm Update Probability

- The Update Probability for False Alarms
  - For Noise Input
  - Uniformly Distributed $\underline{y}$

$$UP_{FA} = SP_{FA} \cdot V_{FA}$$

- When p($\underline{y}$) is Not Uniform
  - More General Analysis Required
  - Use an Integral or Sum to Evaluate UP_{FA}

Mean Change in Score Function for False Alarm Update

- Update Criteria is

$$\underline{e}^T \cdot E^{-1} \cdot \underline{e} \le R_0^2$$

- Probability of a False Alarm Meeting This Criteria is UP_{FA}
- Mean score Function Increment is

$$Exp\left\{dL \middle| H0\right\} = \left(1 - UP_{FA}\right) \cdot \ln\left(\frac{1 - EP_D}{1 - UP_{FA}}\right)$$

$$+ UP_{FA} \cdot \left(\ln\left(EP_D\right) - \tfrac{M}{2}\ln\left(2\pi\right) - \tfrac{1}{2}\ln\left(\left|E\right|\right) - \tfrac{1}{2}\,\tfrac{M}{M+2} \cdot R_0^2\right)$$

System Detection Function

- From The Specified System
  - Probability of Detection SSP_D
  - Probability of False Alarm SSP_{FA}
- Define Two Thresholds for the score Function
  - Upper Threshold for Declaring Detections
  - Lower Threshold for Dropping Tracks
- Upper Threshold

$$T_D = \ln\left(\frac{SSP_D}{SSP_{FA}}\right)$$

- Lower Threshold

$$T_F = \ln\left(\frac{1 - SSP_D}{1 - SSP_{FA}}\right)$$

Track File Pruning

- Measures Taken to Eliminate Probable Noise Tracks
  - – Decrease Number of Noise Tracks
  - – The Association Limit is an Example
- Effect Is
  - – Control Computational Requirements
  - – Decrease Probability of Detection
  - – Decrease Probability of False Alarm
  - – Adjust T_D to Compensate

MHT Performance Measures

- System Performance Specifications
  - Probability of Detection SSP_D
  - Probability of False Alarm SSP_{FA}
- Time TAD from First Hit to Detection
- Mean Life of Noise Tracks TDT
- Number of Noise Tracks NNT Carried at One Time
  - Mean
  - Peak

Mean Change in Score Function

- Mean Change in score Function
  - Per Dwell, Frame, or Look
  - Used in Approximations for Means of
    - TAD, latency of tracker in declaring detections
    - TDT, mean life of a target track
    - NNT, number of noise or clutter tracks
- Needed For
  - Target Tracks
  - Noise Tracks

MHT Trade Space

- M – Number of Measurements Used in Association
- E – Tracker Error Covariance Matrix
- |E| – Determinant of E
- R_0^2 – Association Limit
- P_D – Probability of Detection
- EP_D – Value of P_D for which MHT is Tuned
- SP_{FA} – Differential Probability $p(\underline{y}|H0)\cdot d\underline{y}$

MHT Design Process Goals

- Determine Tradeoffs Between
  - SSP_D
  - SSP_{FA}
  - Other
- Interact with
  - System Requirements Definition
  - Signal Processor Requirements Definition
- Define a Compliant MHT Design

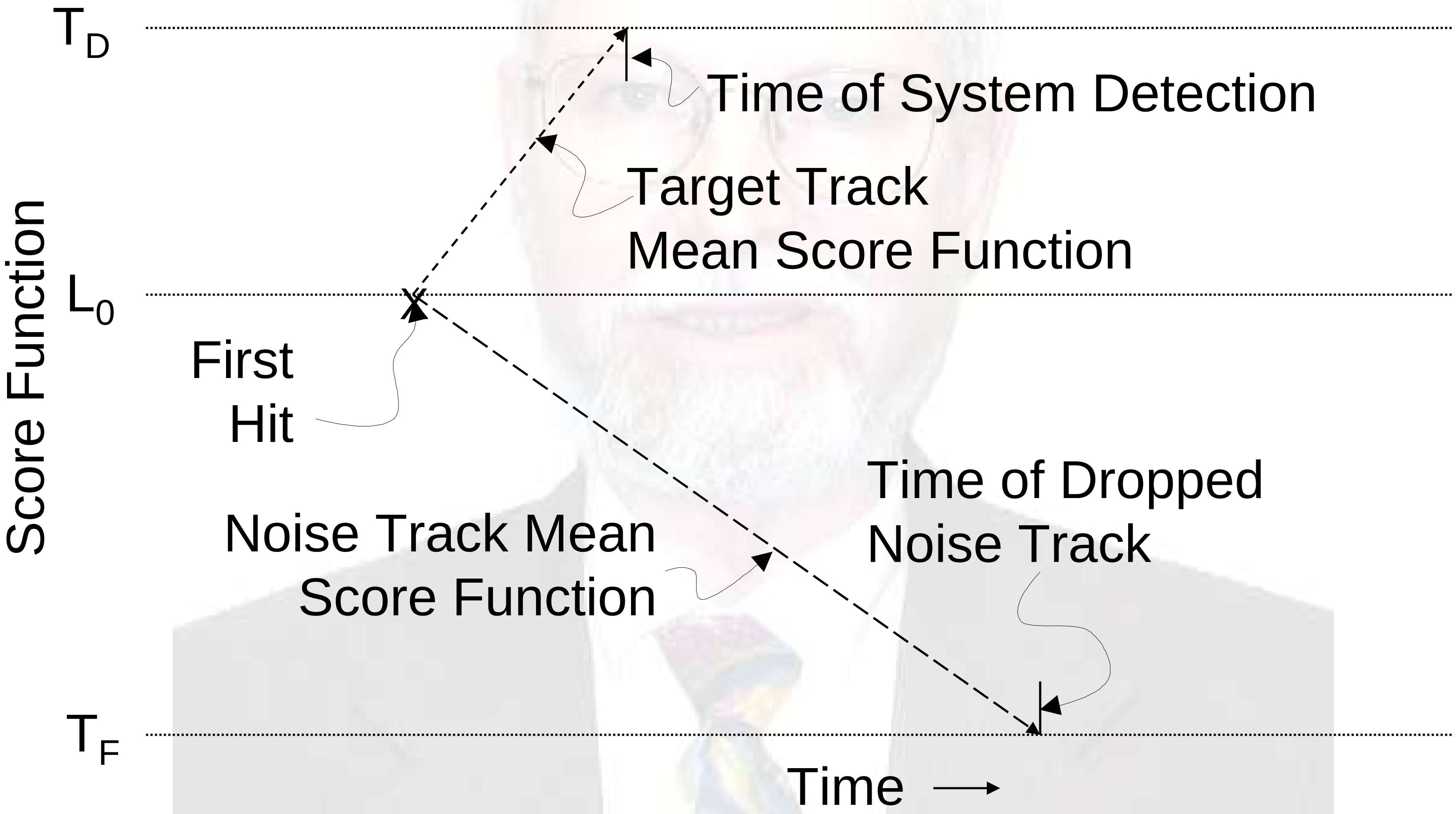
MHT Design Alternatives 1

- In association logic
  - – Omit some measurement parameters
  - – Decrease M
- Decrease |E| by modifying the radar mode design
  - – Decreasing tracker revisit time
  - – Increasing measurement accuracy
- Change or eliminate association limit

MHT Design Alternatives 2

- System Requirements
  - Modify MDL to change P_D and EP_D
  - Change SP_{FA} by changing the signal processor detection threshold
- MHT Design – Add Track File Pruning
- Use More Signal Processor Data
  - Estimate P_D from SNR
  - Get better agreement between P_D and EP_D

MHT Score Function vs. Time



T_D

Time of System Detection

Target Track
Mean Score Function

Score Function

L_0

First
Hit

Noise Track Mean
Score Function

Time of Dropped
Noise Track

T_F

Time →

The Performance Parameters

- Mean Time from Target Appearance to System Detection, in Look Times is

$$TAD = \frac{1}{P_D} + \frac{T_D - L_0}{\mathrm{Exp}\{dL | H1\}}$$

- Mean Life of Noise Track in Look Times

$$TNT = \frac{L_0 - T_F}{-\mathrm{Exp}\{dL | H0\}}$$

- N_{FA} is the Mean Number of False Alarms per Look
- Mean Number of Noise Tracks

$$NNT = N_{FA} \cdot TNT$$

Estimating the Mean Life of a Target Track

- TDT is the Mean Time that a True Declared Target Track will Take to be Incorrectly Dropped as a Noise Track Due To Missed Detections
- The Number of Missed Detections K Required to Drop a True Target Track

$$K = \frac{T_D - T_F}{\ln\left(1 - UP_{FA}\right) - \ln\left(1 - P_D\right)}$$

- Probability that This Will Happen

$$P_{DT} = \left(1 - P_D\right)^K \approx \exp\left(-K \cdot P_D\right)$$

Mean Life of a Target Track

- Once K Looks Have Passed
  - Probability of a Drop Track is Same for Every Succeeding Look
  - Geometric or Poisson Distribution
  - Mean is TDT

$$TDT = \frac{\left(1 - P_{DT}\right)}{P_{DT}}$$

- Excellent Approximation for Small P_{DT}

$$TDT \approx \exp\left(K \cdot P_D\right)$$

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Topic 18: Log Polar Tracker Coordinate Systems

Sensor Systems Engineering for the 21st Century

Coordinate Systems

- Variance Model
  - Coordinate System Irrelevant
  - No Nonlinear Recursion
  - Use Simplest Possible formulation
- Nonlinear Recursion
  - Coordinate System Critical
  - Minimize Sensitivity of Elements of H
  - Rationalize *Ad Hoc* Convergence Measures
- You can Switch Coordinate Systems In A Run

Log Polar Coordinate System

- Use with Single Platform
- To Merge Multiple Platform Data
  - Pass State Vector and Covariance
  - Sufficient Estimator Defines Data Requirements for Fusion
- State Vector

$$\underline{x} = \begin{bmatrix} \ln\left(\dfrac{R}{R_c}\right) \\ \underline{u}_r \end{bmatrix}, \quad \underline{u}_r = \frac{\underline{r}}{R}$$

Analytic Function Rationale

- Logarithm of Range Vector

$$\ln\left(\frac{\underline{r}}{R_c}\right)=\ln\left(\frac{R}{R_c}\right)+\frac{\pi}{2}\cdot\underline{u}_r$$

- Notation Definition

$$\rho=\ln\left(\frac{R}{R_c}\right)$$

Estimating Three States

- Four States
  - Log Range
  - Unit Line of Sight Vector $\underline{u}_r$
- Eliminate One Direction Cosine
  - Eliminate Largest Direction Cosine
  - Remember Its Sign
- Keep Smallest Two Direction Cosines
  - Most Linear in Estimation
  - Don’t Get Near $\underline{+}$ 1

Formulation of Log Polar

- Platform Moves, Target May Move
  - States Computed from NED Range Vector at Specific Epoch (Target x, Own Ship p)

$$\underline{r}_0 = \underline{x}_0 - \underline{p}_0$$

  - Measurements In Terms of Range Vector at Each Measurement Epoch

$$\underline{r}_i = \underline{x}_i - \underline{p}_i = \underline{r}_0 + \Delta\underline{r}_i$$

$$\Delta\underline{r}_i = \left(\underline{x}_i - \underline{x}_0\right) - \left(\underline{p}_i - \underline{p}_0\right)$$

Pros and Cons of Log Polar

- Pros
  - Mathematical Basis in Analytic Function Theory Simplifies Gradients
  - No Singularities Near Locus of Convergence
  - Sensitivity of H Matrix to Errors in Initialization Minimized
- Cons
  - Not Simplest for Gradient Equations
  - Origin Moves with Time

References

- Modeling Rotating Bodies with Quaternions, J.K. Beard, PDF file available from http://www.jameskbeard.com
- "An Efficient, Reliable Approach for Hydroacoustical Bearings Only Target Motion Analysis," Proceedings IEEE Conference on Decision and Control, 14 December 1984, PP 1251-1256
- “Mathematical Techniques in Multisensor Data Fusion,” David L. Hall, Artech (1992)

Numerical Issues in Kalman Filters

- The Reality
  - Covariance matrices are ill conditioned on data update in most practical Kalman filters
  - Result can be a matrix that is not positive definite due to numerical errors
  - This problem prevented wide use of Kalman filters from 1960 until about 1975
- Fixes Were Several Covariance Update Procedures

Covariance Update Procedures

- Stabilized Form

$$P = \left(I - K \cdot H\right)^T \cdot \tilde{P} \cdot \left(I - K \cdot H\right) + K \cdot R \cdot K^T$$

- Divide, Not Subtract, Approach

$$P^{-1} = \tilde{P}^{-1} + H^T \cdot R^{-1} \cdot H$$

- Holistic Approach: Square Root Filters
  - Represents covariance matrix as a Cholesky factorization or similar “square root”
  - Other features support better numerical performance than Kalman filter

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Topic 19: Square Root Filters

Sensor Systems Engineering for the 21st Century

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Topic 19: Square Root Filters

Sensor Systems Engineering for the 21st Century

Square Root Kalman Filters

- What Is a Square Root Kalman Filter?
  - Square Root Nomenclature Based On

    $$P = S \cdot S^T$$

  - Variable Changes Used to Eliminate P from Extrapolation and Update Equations in Favor of S
- Reasons to Use Square Root Filters
  - When Some States are Poorly Observable, Variance Ratios are Large
  - Variance Ratios are Reflected as Large Dynamic Range in Covariance Matrix
  - Consequential Numerical Problems are Avoided

References

- Astronautical Guidance, R. H. Battin, McGraw-Hill (1964) documents Potter’s introduction of the square root filter.
- Filtering for Stochastic Process with Applications to Guidance, R. S. Bucy and P. D. Joseph, John Wiley & Sons, NY (1968)
- Agee,, W. S., and Turner, R. H., “Triangular decomposition of a positive definite matrix plus a symmetric dyad with application to Kalman filter,” White Sands Missile Range Tech. Rep. No. 38 (1972).
- Carlson, N. A., “Fast triangular factorization of the square root filter,” AIAA J. 11, no. 9, pp 1259-1265 (1973)
- Solving Least Squares Problems, C. L. Lawson and R. J. Hanson, Prentice-Hall, Englewood Cliffs, NJ (1974).
- Factorization Methods for Discrete Sequential Estimation, G. J. Bierman, Academic Press (1977).

Types of Square Root Filters

- Square Root Covariance Filter (Potter) Using S
- Square Root Information Filter (Lawson & Hanson, Bierman) Using Inverse of S
- Modified Square Root Covariance Filter (“UDUT”, Agee, Turner, Carlson, Bierman)
- Non Uniqueness Theorem
  - If $S \cdot S^T = P$
  - T is any Orthogonal Matrix $T \cdot T^T = I$
  - Another Matrix $S' = S \cdot T$
  - Then $S' \cdot S'^T = P$

Factorization Techniques

- Triangularization using Householder Transformations
  - Useful in Solving Overdetermined Least Squares Problems
  - SRIF Maps Kalman Filter to Two Such Problems
  - Batch Estimators Use This Approach
- Modified Weighted Gram-Schmidt Orthogonalization
  - Can Be Used to Perform UDUT Factorization
  - Used in Covariance Extrapolation in UDUT Square Root Filter
- Cholesky Factorization
  - Simple Identity
  - Maps Initial Covariance Matrices to Initial Square Root Matrices
- Singular Value Decomposition
  - Not Used in Square Root Filters
  - Useful for Analysis of Covariance Matrices and Measurement Sensitivity Matrices

The Householder Transformation

- Definition

$$T = I - 2 \cdot \underline{u} \cdot \underline{u}^T, \ \ \underline{u} \text{ any unit vector}$$

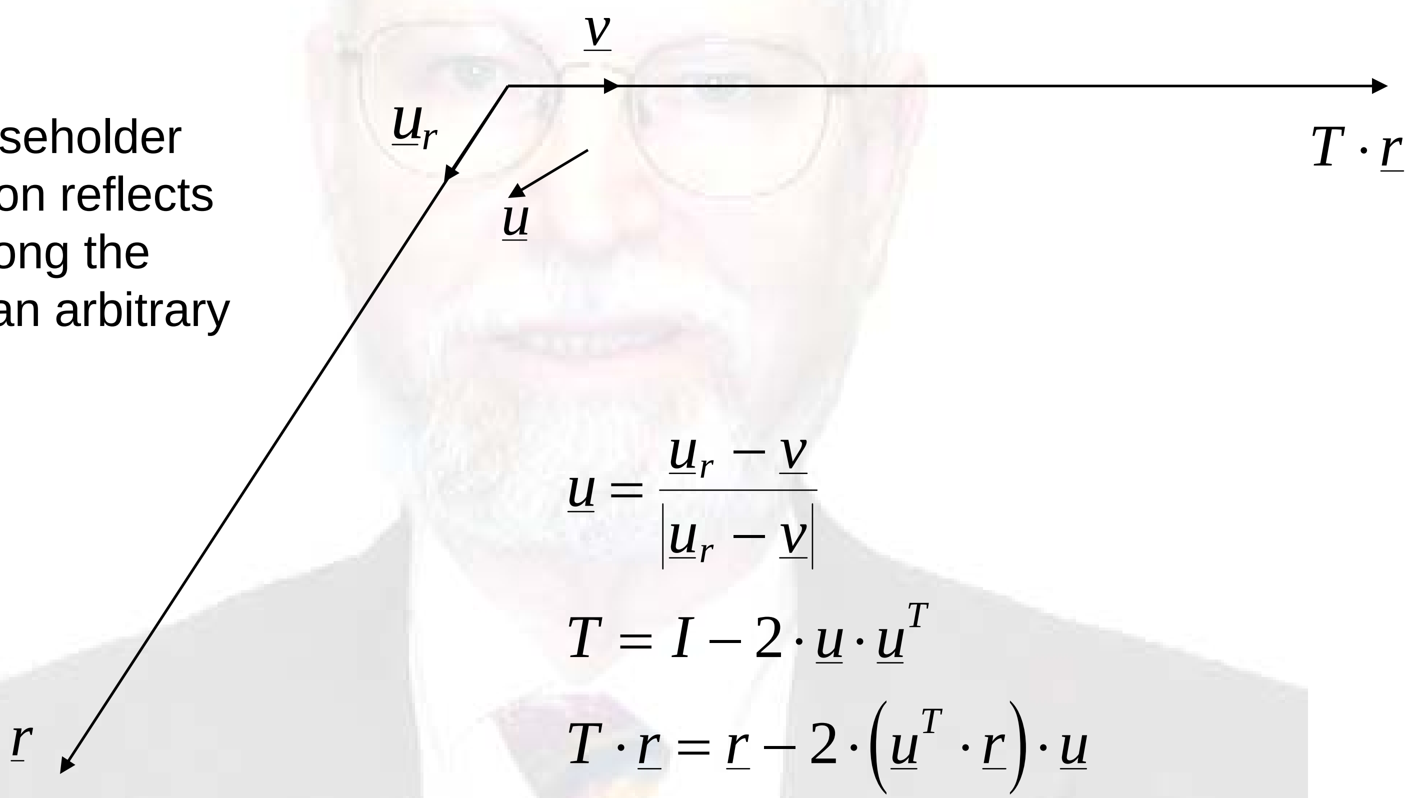
- Properties
  - Matrix Operator

$$\underline{x} = T \cdot \underline{y} = \underline{y} - 2 \cdot \left(\underline{u}^T \cdot \underline{y} \right) \cdot \underline{u}$$

  - Components of $\underline{y}$ Orthogonal to $\underline{u}$ are Not Affected
  - Components of $\underline{y}$ Along $\underline{u}$ are Reversed in Direction
  - Length of $\underline{x}$ is Same as Length of $\underline{y}$
  - T·T=I; T is Own Inverse, or T is Idempotent
  - Sometimes called “Mirror” Because $\underline{x}$ is Reflection in Mirror Plane Normal to $\underline{u}$

Use of Householder Transformation

Here, a Householder transformation reflects a vector r along the direction of an arbitrary vector v



$$\underline{u} = \frac{\underline{u}_r - \underline{v}}{\left|\underline{u}_r - \underline{v}\right|}$$

$$T = I - 2 \cdot \underline{u} \cdot \underline{u}^T$$

$$T \cdot \underline{r} = \underline{r} - 2 \cdot \left(\underline{u}^T \cdot \underline{r}\right) \cdot \underline{u}$$

Annihilation of a Column of a Matrix Below the Main Diagonal

- Basic Principles
  - The Householder transformation will not affect elements of a vector for which corresponding elements of $\underline{u}$ are zero
  - Use elements of $\underline{u}$ corresponding to the main diagonal and below
  - Reflect the column vector marked for annihilation along the axis corresponding to the element on the main diagonal, as in the example on the last slide
- Formulation
  - Column vector $\underline{a}$, $\underline{v}T = [1, 0, 0, ...]$
  - Select $\underline{a} \pm \underline{v}$ with the sign selected to maximize the length
  - Operation $T \cdot A$ is performed in-place without explicitly calculating or storing T

The Normalized Data Equation

- Definition: A Relationship of the Form

$$A\cdot\left(\underline{\hat{x}}-\underline{x}\right)=\underline{v}$$

  - Where $\underline{x}$ is the state vector
  - A has more rows than columns
  - Covariance of noise vector $\underline{v}$ is I
- Solution: Triangularize A with a sequence of Householder transformations to form a new NDE

Transformed NDE

- Sequence of Householder Transformations T

$$T \cdot A \cdot \left(\underline{\hat{x}} - \underline{x}\right) = T \cdot \underline{v}$$

$$\begin{bmatrix} R \\ 0 \end{bmatrix} \cdot \left(\underline{\hat{x}} - \underline{x}\right) = \underline{v'}$$

$$Cov\left\{\underline{\hat{x}} - \underline{x}\right\} = R^{-1} \cdot R^{-T}$$

The Square Root Information Filter Update Equation

- The Update Data

$$\tilde{R}\cdot\left(\underline{\tilde{x}}-\underline{x}\right)=\underline{v}_1,\ \ \tilde{P}=\tilde{R}^{-1}\cdot\tilde{R}^{-T}$$

$$\underline{z}=\underline{a}\left(\underline{\tilde{x}}\right)+A\cdot\left(\underline{\tilde{x}}-\underline{x}\right)+\underline{v}_2,\ \ Cov\left\{\underline{v}\right\}=I$$

- The Update

$$T\cdot\begin{bmatrix}\tilde{R}\\ A\end{bmatrix}\cdot\left(\underline{\hat{x}}-\underline{x}\right)+T\cdot\begin{bmatrix}\underline{0}\\ \underline{z}-\underline{a}\left(\underline{\tilde{x}}\right)\end{bmatrix}=T\cdot\begin{bmatrix}\underline{v}_1\\ \underline{v}_2\end{bmatrix}$$

$$\begin{bmatrix}R\\ 0\end{bmatrix}\cdot\left(\underline{\hat{x}}-\underline{\tilde{x}}\right)+\begin{bmatrix}d\,\underline{\hat{z}}\\ \underline{e}\end{bmatrix}=\underline{v'}$$

Association and the MHT Cost Function

- Residual Error Vector <u>e</u> is

$$\underline{e} = T_{22} \cdot \left(\underline{z} - \underline{a}\left(\underline{\tilde{x}}\right)\right)$$

$$T_{22}^{T} \cdot T_{22} = \left[I + A \cdot \tilde{P} \cdot A^{T}\right]^{-1}$$

- See SRIF Write-up Page 10
- Squared Length is

$$\underline{e}^{T} \cdot \underline{e} = \left(\underline{z} - \underline{a}\left(\tilde{x}\right)\right)^{T} \cdot \left[A \cdot \tilde{P} \cdot A^{T} + I\right]^{-1} \cdot \left(\underline{z} - \underline{a}\left(\tilde{x}\right)\right)$$

Square Root Information Filter Extrapolation Data Equations

- Data Equations for the Extrapolation

$$\left(\underline{\tilde{x}}-\underline{x}\right)=\Phi\cdot\left(\underline{\hat{x}}(-)-\underline{x}(-)\right)+G\cdot\underline{w}$$

$$R_w\cdot\underline{w}=\underline{v}_1,\;\;Q=R_w^{-1}\cdot R_w^{-T}$$

$$R_-\cdot\left(\underline{\hat{x}}(-)-\underline{x}(-)\right)=\underline{v}_2,\;\;P(-)=R_-^{-1}\cdot R_-^{-T}$$

- The First Data Equation is

$$R_-\cdot\Phi^{-1}\cdot\left(\underline{\tilde{x}}-\underline{x}\right)=R_-\cdot\left(\underline{\hat{x}}(-)-\underline{x}(-)\right)$$
$$+R_-\cdot\Phi^{-1}\cdot G\cdot\underline{w}$$

NDE’s for SRIF Extrapolation

- The Data Equations

$$R_w \cdot \underline{w} = \underline{v}_1$$

$$-R_{\_} \cdot \Phi^{-1} \cdot G \cdot \underline{w} + R_{\_} \cdot \Phi^{-1} \cdot \left(\underline{\tilde{x}} - \underline{x}\right) = \underline{v}_2$$

- The Augmented Data Equation

$$T \cdot \begin{bmatrix} R_w & 0 \\ -R_{\_} \cdot \Phi^{-1} \cdot G & R_{\_} \cdot \Phi^{-1} \end{bmatrix} \cdot \begin{bmatrix} \underline{w} \\ \underline{\tilde{x}} - \underline{x} \end{bmatrix} = T \cdot \underline{v}$$

$$\begin{bmatrix} \tilde{R}_w & \tilde{R}_{wx} \\ 0 & \tilde{R} \end{bmatrix} \cdot \begin{bmatrix} \underline{w} \\ \underline{\tilde{x}} - \underline{x} \end{bmatrix} = \underline{v}'$$

Weighted Gram-Schmidt Orthogonalization

- Definition

  Gram-Schmidt orthogonalization of a matrix W is the operation of "squaring up" the rows (or columns) of a matrix so that they are all orthogonal to each other. The resulting matrix V has orthogonal rows (or columns), but the lengths of the rows are not normalized. The matrix W need not be square.

- Definition of Weighted Orthogonality

$$\underline{w}_i^T \cdot D \cdot \underline{v}_k = 0$$

- Beginning with Row N,

$$\underline{v}_i = \underline{w}_i - \sum_{k=i+1}^{N} \frac{\underline{w}_i^T \cdot D \cdot \underline{v}_k}{\underline{v}_k^T \cdot D \cdot \underline{v}_k} \cdot \underline{v}_k = \left[\prod_{k=i+1}^{N} B_{Dvk} \right] \cdot \underline{w}_i$$

Weighted Subspace Operator

- The Operator

$$B_{Dvk} = I - \frac{\underline{v}_k \cdot \underline{v}_k^T \cdot D}{\underline{v}_k^T \cdot D \cdot \underline{v}_k}$$

- Result of Operation is Orthogonal to $\underline{v}_k$
- Thus Rows of V are Orthogonal
  - Orthogonality Definition Uses Weighting D
  - Rows are Not Normalized in Length

WGSO as a Matrix Factorization

- Rearrange the Orthogonalization

$$\underline{w}_i = \underline{v}_i + \sum_{k=i+1}^{N} \frac{\underline{w}_i^T \cdot D \cdot \underline{v}_k}{\underline{v}_k^T \cdot D \cdot \underline{v}_k} \cdot \underline{v}_k$$

- An Equivalent Operation

$$W = U \cdot V$$

- Matrix U Defined by Rearranged Equation

The Matrix Factorization

- The Matrix U

$$u_{ik}\begin{cases} = 0, \;\; i < k \\ = 1, \;\; i = k \\ = \dfrac{\underline{w}_i^T \cdot D \cdot \underline{v}_k}{\underline{v}_k^T \cdot D \cdot \underline{v}_k}, \;\; i > k \end{cases}$$

- The Orthogonality of V

$$V \cdot D \cdot V^T = D_+, \quad W \cdot D \cdot W^T = U \cdot D_+ \cdot U^T$$

WGSO in Covariance Extrapolation

- Covariance Matrix Factorization

$$P = U \cdot D \cdot U^T$$

- Covariance Extrapolation

$$\tilde{U} \cdot \tilde{D} \cdot \tilde{U}^T = \Phi \cdot U_- \cdot D_- \cdot U_-^T \cdot \Phi^T + G \cdot Q \cdot G^T$$

$$W = \left[\Phi \cdot U_-, G \right]$$

$$D = \begin{bmatrix} D_- & 0 \\ 0 & Q \end{bmatrix}$$

The UDUT Square Root Filter

- Extrapolation
  - State Vector and Transition Matrix are the Same as the EKF
  - WGSO Used for Covariance Extrapolation
- Update
  - A complex series of algebraic identities is used to update the state vector and covariance matrix one measurement at a time
- Association and MHT Cost Function
  - No Change from the EKF
  - Quadratic Form Saved from Association for MHT Cost Function

UDUT Update

- By Agee, Turner, Carlson, and Bierman
  - Given

$$\tilde{P} = \tilde{U} \cdot \tilde{D} \cdot \tilde{U}^T, \quad P = \left(I - K \cdot H\right) \cdot \tilde{P}$$

$$P = \hat{U} \cdot \hat{D} \cdot \hat{U}^T$$

  - Find $\hat{U}, \ \hat{D}$
- Requirements
  - Efficiency and Robustness
  - P Must Not Be Computed

AGTB Update

- Initialization

$$\underline{f} = U^T \cdot H^T, \quad \underline{v} = D \cdot \underline{f}$$

$$c_1 = r + \tilde{d}_1 \cdot f_1^2, \quad \hat{d}_1 = \frac{r}{c_1} \cdot \tilde{d}_1, \quad \underline{k}_2 = v_2 \cdot \underline{\tilde{u}}_2$$

- Recurrence for j=2 to N

$$c_j = c_{j-1} + \tilde{d}_j \cdot f_j^2, \quad \hat{d}_j = \tilde{d}_j \cdot \frac{c_{j-1}}{c_j}$$

$$\underline{\hat{u}}_j = \underline{\tilde{u}}_j - \frac{f_j}{c_{j-1}} \cdot \underline{k}_j, \quad \underline{k}_{j+1} = \underline{k}_j + v_j \cdot \underline{\tilde{u}}_j$$

Other Quantities Available

- Kalman Gain

$$K = \frac{k_{N+1}}{c_N}$$

- Variance of Innovations Sequence

$$H \cdot \tilde{P} \cdot H^T + r = c_N$$

Pros and Cons of the SRIF

- Pros
  - Excellent numerical performance in the most difficult applications
  - Low operation count
  - Can add and drop states according to observability without adding complexity to the algorithm
  - Simple software modules
  - The association and MHT cost function computation is built-in
  - Uses all the measurements at once
  - Can share formulation with batch estimators and sensor variance models (see File of the Week)
- Cons
  - Association requires almost complete update process
  - Covariance computation must be added to update process for interfacing to displays and links to command and control

Pros and Cons of the UDUT Square Root Filter

- Pros
  - Excellent numerical performance in the most difficult applications
  - Simple software modules
  - Low operation count
  - Easily applied to existing Kalman filters by substituting modules
- Cons
  - Association process requires EKF procedure with no commonality; once computed, the quadratic form from the association calculation is used in the MHT cost function, as in the EKF
  - Adding or dropping states in real time can be done only at expense of added complexity
  - Updates are done using only one measurement at a time
  - Covariance computation must be added to update process for interfacing to displays and links to command and control

SRIF Modules

- SRIFEXT
  - State vector extrapolation
  - Covariance (square root information) extrapolation
- SRIFDAT
  - State vector update
  - Square root information update
- TSOLVER, PCOMP, CHOLES, RINV, etc.
  - Utilities to compute $\Delta \underline{x}$
  - Utilities to monitor tracker observabilities

UDUT Modules

- UDFACT – U-D Factorization
- MWGS
  - Modified Weighted Gram-Schmidt Orthogonalization
  - UDUT Covariance Extrapolation
- UDUPDT – UDUT Update
- PDIAG, UDUTP, Etc.
  - Utilities to support displays and controls
  - Utilities to monitor tracker performance

ATEP SYS12525

Radar Trackers and Applications for SAADS

November 2, 1999

Topic 20: Sensor Variance Models

Sensor Systems Engineering for the 21st Century

File of the Week

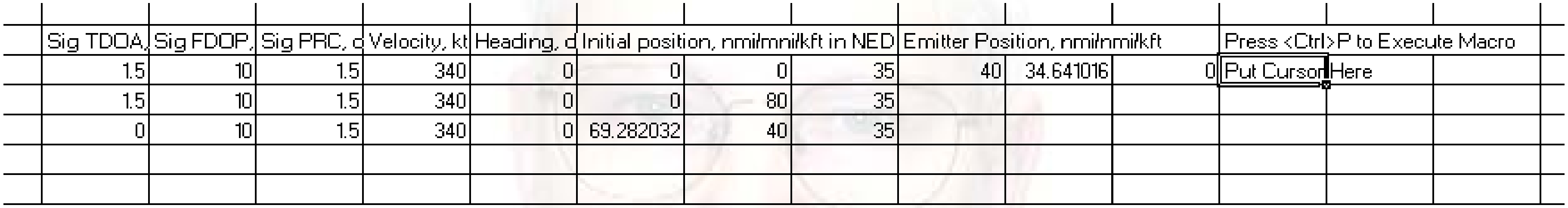
- A Sensor Variance Model
  - Models a Scenario
    - Up to 10 Aircraft Flying in Straight Lines
    - One Emitter Position
    - Up to 3 Passive Sensors on Each Aircraft
      - Time Difference of Arrival (TDOA)
      - Frequency plus Doppler
      - Phase Rate of Change (PRC)
  - Produces Cramer-Rao Bound as Output
- Uses SRIF Formulation

Mac Users

- Current Release of Office 97 Fixes Bug
- Moving an Excel Macro Between Platforms
  - Modifies “DefDbl A-H, O-Z” to "DefDbl A-J, Q-X"
  - DefInt line
    - Also Modified
    - Becomes Invisible
- Sample Lines from Comments
  - Copy Over Bad Variable Typing Lines
  - Delete Comment Mark in Column 1

Due to incompatibility between Releases of Microsoft ® Excel ™, these files are not provided.

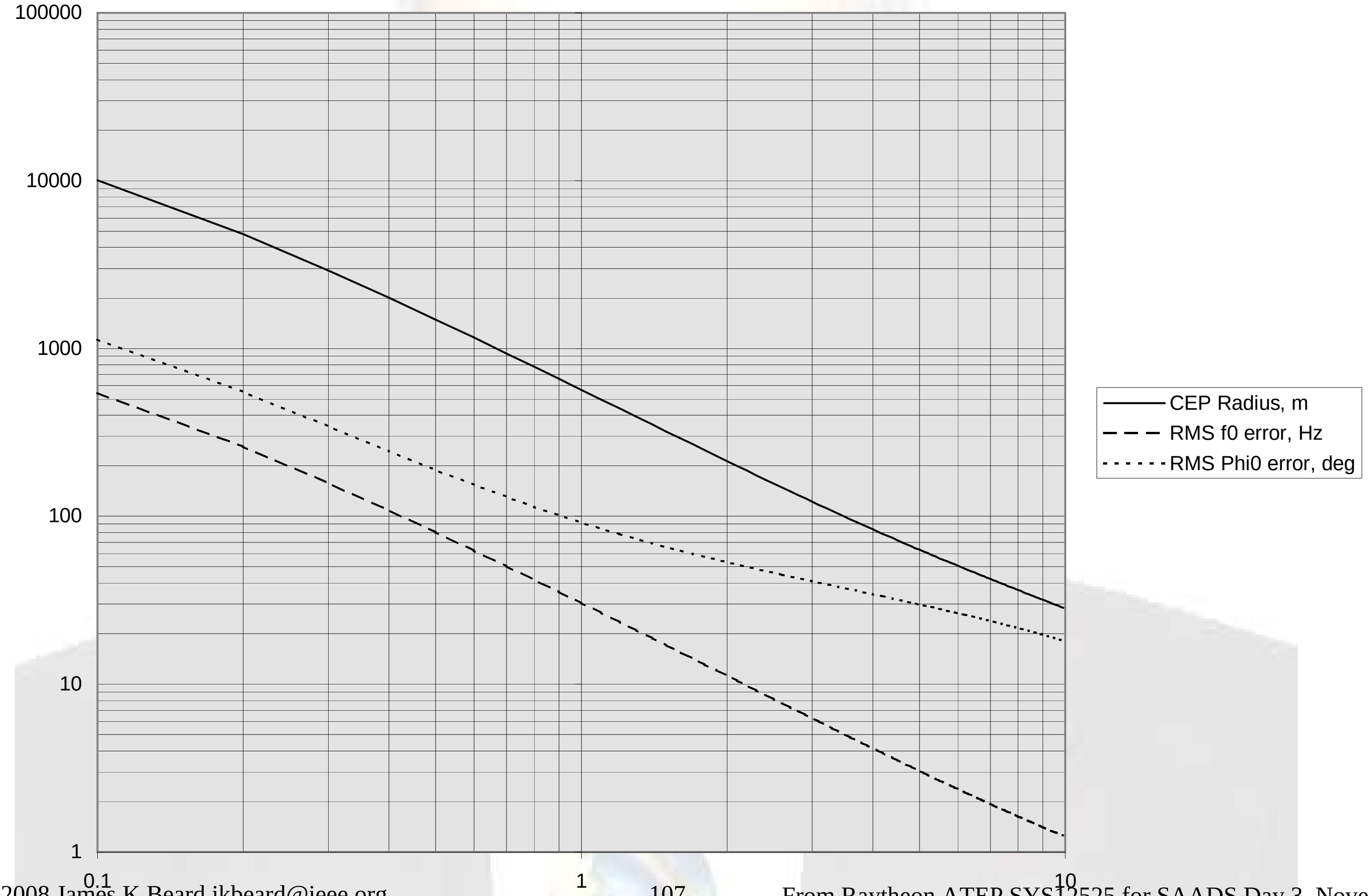
The Worksheet



	Sig TDOA,	Sig FDOP,	Sig PRC, c	Velocity, kt	Heading, d	Initial position, nmi/mni/kft in NED			Emitter Position, nmi/nmi/kft			Press <Ctrl>P to Execute Macro		
	1.5	10	1.5	340	0	0	0	35	40	34.641016	0	Put Cursor Here		
	1.5	10	1.5	340	0	0	80	35						
	0	10	1.5	340	0	69.282032	40	35						

- Inputs
  - Sensor Variances
  - Aircraft Velocity, Heading, Position
  - Emitter Position
- To Run
  - Position Cursor as Indicated
  - Press <Ctrl>P

Typical Output



ATEP SYS12525

Radar Trackers and Applications for SAADS

November 2, 1999

Topic 21: ESA Radar Schedulers and Resource Managers

Sensor Systems Engineering for the 21^{st} Century

System Timing Concepts

- System Frames
  - Period of Revisit of Targets in Track
  - Subframes – Timing of Waveforms
- Definitions
  - Major Frame
    - Target Revisit Time
    - Time Between Dwells or Updates
  - Subframe
    - Target Dwell Time
    - Waveform Duration

Fundamental ESA Paradigm

- ESA Radar is Multiple Virtual Radars
  - Each Time Shares Access to Antenna
  - Antenna is Independently Steered
  - Virtual Radar Modes are Independent
- ESA vs. Mechanically Steered Field of View
  - Field of Regard
    - What Can Be Seen with Mechanical Steering
    - Changes with Platform Rotation
  - Field of View
    - What Can Be Seen Instantaneously
    - Without ESA, Single Beam
    - With ESA, Total Coverage Over Steering Range
    - Changes with Platform Rotation

Determination of Frame Time

- Factors Include
  - Maximum Allowable Variance at Association
    $T_{FRAME} \cdot q' < \text{maximum allowable variance}$
  - Determined
    - Electronically with ESAs
    - Mechanically Scanned
      - Ground Search Radars – Sweep Time
      - Gimbaled Antennas – Search or Raster Scan Time
- Subframes Within Frames
  - Time Dedicated to Single Function or Target
  - Sensor Mode Varies Between Subframes

Determination of Subframe Time

- Definition
  - A subframe is the time within a frame which is dedicated to a single target or function
  - Sensor functions supported in subframes
    - Search
    - Verification
    - Tracking
    - Other
  - Sensor mode in one subframe may be different form the mode in other subframes
- Sum of
  - Beam steering time
  - Propagation or fill time (2·Rmax/c)
  - CPI or FFT Integration time
  - Other as Required by System

Frame Timing Considerations

- Track While Scan Sensors
  - Subframe Time
    - Beam or Pixel Dwell Time
    - Not Necessarily Synchronous
  - True Of
    - Most EO/IR Sensors
    - Most Passive RF Sensors
    - Many Active Radars
- Tradeoffs
  - Increasing Subframe Time
    - Increases Sensitivity and Maximum Range
    - Decreases Coverage Rate
  - Decreasing Subframe Time
    - Increases Coverage Rate
    - Increases Number of Dwells per Major Frame

The System Clock

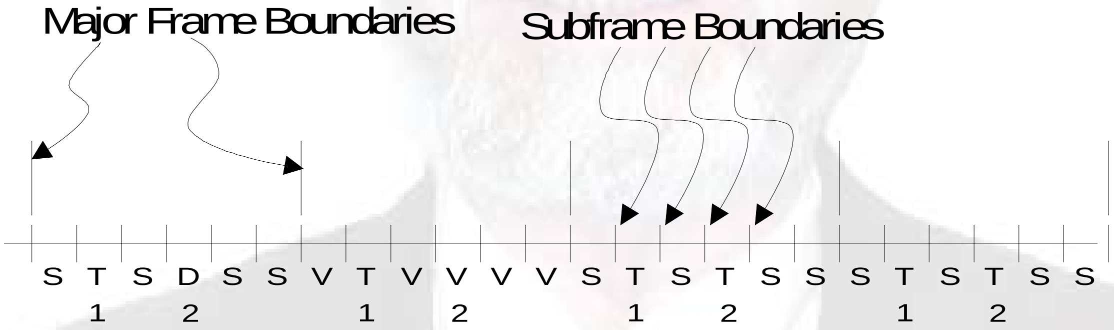
- Only One is Used
  - Every Clock Causes EMI Problems
  - Less is Better, One is Best
- Counted Down from System Clock
  - All PRI’s
  - Pulse Lengths
  - Subframe times
  - Frame times
- Ultimate System Granularity

Subframe Function or Mode Assignments

- Each Subframe
  - Mode Assigned Independently
  - In an ESA System
    - » Assigned to a particular target in track
    - » Allocated to control tracker revisit time
  - Simple Beam Dwell in Other Systems
- Foreground-Background Prioritization
  - Highest: Track
  - Next: Verify
  - Background: Search
  - Lowest: Idle

Allocation of Radar Resources

- Each Subframe Allocated Separately
  - Search
  - Tracking Functions
    - Verification/Track Initialization
    - Updates
  - Other
- Sensor Time Line



LEGEND
T Track target
S Search
D New detection
V Verify