Physics, Chemistry, and Mathematics of Photography

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Topics

• Part I: Resolution
  – On the film
  – On the print
  – On the slides
  – In television

• Part II: Color, shading, and prints
  – Contrast, color, and the Zone System
  – Lens design
  – Digital and conventional photography
  – The portal: scanning and scanners
Optical Fourier Transform

Light Mask Lens

Focused Light Fourier Plane

\[ \lambda \]
Optical Fourier Transform

- Collimated monochromatic impinging light
- Mask modulates light intensity
- Lens makes all effective path lengths to focal point equal
- Light at focal point is

\[ a(\rho, \varphi) = \frac{D_0}{2} \int_{0}^{\pi} \int_{-\pi}^{\pi} m(r, \theta) \cdot \exp \left( -j \cdot \frac{2\pi}{\lambda} \cdot r \cdot \rho \cdot \cos(\theta - \varphi) \right) \cdot r \cdot dr \cdot d\theta \]

\( r, \theta \leftrightarrow \text{Lens Plane}; \ r \text{ is distance} \)

\( \rho, \varphi \leftrightarrow \text{Fourier Plane}; \ \rho \text{ is half-cone angle} \)

“Electro-Optical Systems Analysis,”
K. Seyrafi, p 174-177
The Airy Disk

- An open, uniformly weighted circular aperture of diameter $D_0$
- Intensity on the Fourier plane is

$$a(\rho) = \frac{\pi \cdot D_0^2}{4} \cdot \frac{2 \cdot J_1 \left( \frac{\pi \cdot D_0 \cdot \rho}{\lambda} \right)}{\pi \cdot D_0 \cdot \rho \cdot \frac{\lambda}{\rho}}$$

- Resolution is peak-to-null distance

$$\Delta \rho = \frac{1.22 \cdot \lambda}{D_0}$$

The Diffraction Limit on the Focal Plane

• The f/stop or f-number is
\[(f / no) = \frac{f}{D_0}\]

• The numerical aperture for a lens is
\[NA = n' \cdot \sin (u') = \frac{1}{2 \cdot (f / no)}\]

• Resolution distance is
\[\Delta \rho \cdot f = 1.22 \cdot (f / no) \cdot \lambda = \frac{0.61 \cdot \lambda}{NA}\]
• A resolvable line is two resolution elements
  – Two lines must have a resolvable space between them
  – The distance between the line and the resolvable space is a resolvable element
• The number of lines per millimeter is

\[ lpmm = \frac{1}{2 \cdot \Delta \rho \cdot f} = \frac{1}{2.44 \cdot (f / no) \cdot \lambda} = \frac{NA}{1.22 \cdot \lambda} \]
Resolution on 35 mm Film

• Film resolution is 80 to 200 lines per mm

• Optics limit
  – Diffraction limit is about 133 lines per mm at f/5.6
  – Achieved in laboratory: 80 to 100 lines per mm
  – Achieved by the photographer: 40-80 lines per mm

Practical limit is 40 – 80 lines per mm
Resolution on 70 mm Film

• Film resolution is 80 to 200 lines per mm

• Optics limit
  – Diffraction limit is about 65 lines per mm at f/11
  – Achieved in laboratory: 45 to 80 lines per mm
  – Achieved by the photographer: 25-50 lines per mm

Practical limit is 25 – 50 lines per mm
Number of Pixels

• In 35 mm film
  – Image is 24 mm by 36 mm
  – 4 X 3 aspect limit is 24 mm X 32 mm
  – 4.7 M at 40 lines per mm (M = 1024^2 pixels)
  – 18.75 M at 80 lines per mm

• In 70 mm film
  – Image is 6 X 7 cm
  – 22.4 M in 5.25 X 7 cm at 40 lines per mm
Print Resolution

• Eye is about a 24 mm focal length
• F-stop of pupil is f/2.8 to f/32
  – Typical pupil is f/11 in bright light
  – Eye resolution is 0.3 milliradians for green light
• Print resolution
  – At two feet viewing distance
  – About 144 dots per inch
Print Resolution vs. Distance

[Graph showing the relationship between print resolution (dots per inch) and viewing distance (feet).]
Scanning Slides and Negatives

• Resolution
  – Scale factor from lines per mm to DPI
    \[ \frac{25.4}{\text{in}} \cdot 2 \frac{\text{dots}}{\text{line}} = 50.8 \frac{\text{dots}}{\text{in}} \]
  – 80 line per mm is 4000 DPI

• Vendors of slide/negative scanners
  – SmartDisk: 2700 or 3600 DPI
  – Nikon Coolscan, Canoscan, Microtek, Polaroid: 4000 DPI
  – Minolta Dimage: 2820, 4800 DPI
NTSC Color TV

• RGB is encoded for transmission
  – Illuminance, weighted combination
  – I and Q channels to carry color
  – Signal is compatible with pre-color TV

• Resolution
  – Limited by channel bandwidth to 220 lines
  – Color is less; relationship is complex
## The Encoding Matrix

<table>
<thead>
<tr>
<th></th>
<th>$E_R'$</th>
<th>$E_G'$</th>
<th>$E_B'$</th>
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<tbody>
<tr>
<td>$E_Y'$</td>
<td>0.30</td>
<td>0.59</td>
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<tr>
<td>$E_I'$</td>
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<td>-0.32</td>
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<td>$E_Q'$</td>
<td>0.21</td>
<td>-0.52</td>
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</table>
NTSC Color Encoding

\[ \begin{bmatrix} E_R' - E_Y' \end{bmatrix} \]

1.14

\[ E_C' \]

I Channel BW: 2 MHz
Q Channel BW: 0.5 MHz

\[ \begin{bmatrix} E_B' - E_Y' \end{bmatrix} \]

2.03

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NTSC Summary

• Resolution
  - Limited by TV channel bandwidth
  - About 200 by 480
  - Color is less bandwidth – complex relationship
  - Eye sees 200 by 480
  - Frame averaging with motion enhances perception

• Color
  - Purity and quality are not a problem
  - Blue is lower resolution than red, green
HDTV

• Information given here is from
  - http://www.ee.washington.edu/conselec/CE/kuhn/hdtv/95x5.htm

• Aspect ratio remains 16:9

• Resolution – either of
  - 1280 X 720 (1 MPX)
  - 1920 X 1080 (2 MPX)
# Field of View

<table>
<thead>
<tr>
<th>Format</th>
<th>Field of View, Degrees</th>
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<tbody>
<tr>
<td>TV</td>
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<td>HDTV 1</td>
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<tr>
<td>HDTV 2</td>
<td>30</td>
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<tr>
<td>35 mm</td>
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Viewing Image Size vs. Distance

![Graph showing the relationship between viewing distance and image size for TV, Poster, HDTV 1, HDTV 2, and 35 mm.](image-url)
Viewing Image Size vs. Distance

- Movie
- Drive-In

<table>
<thead>
<tr>
<th>Viewing distance, feet</th>
<th>Width, inches</th>
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<td>1200</td>
<td>6000</td>
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Comparing Digital and 35 mm Focal Plane Resolution

Focal plane, MPX

Equivalent 35 mm lines per mm

- Graphics arts lab
- Best studio work
- Best hand-held
- 3 MPX digital
- 1 MPX HDTV

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References

• HDTV
  – http://www.ee.washington.edu/conselec/CE/kuhn/hdtv/95x5.htm

• Foveon digital photography focal planes
  – http://www.foveon.com/
  – http://www.sigma-photo.com/

• Popular Photography Magazine
  – http://www.popphoto.com/